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Design-based research on instructional lessons for differential calculus incorporating Desmos

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Abstract

Malaysian students have experienced considerable challenges in mastering calculus in recent years, with over 40% failing to attain a credit grade in mathematics. While technology like Desmos can enhance understanding, educators need insights into how students develop conceptual knowledge in technology-based environments. Therefore, this study investigated the pre-university science students' understanding of differential calculus using the Action-Process-Object-Schema (APOS) theory through a Desmos instructional lesson. A design-based research (DBR) strategy was employed to facilitate the real-time design, implementation, and refinement of the intervention. Classroom observations and document analysis revealed that most students achieved the Process phase of understanding based on APOS. The students' interviews also indicated that they found the Desmos-based lesson engaging and valued the opportunity for group discussion. Nevertheless, some students still encountered difficulties entering equations in Desmos. Further refinements of the instructional design were then conducted through a comparative retrospective analysis between the hypothetical learning trajectory and actual learning outcomes. Overall, this study contributes to the existing literature by providing detailed, stage-specific analyses of the design, implementation through a teaching experiment, and retrospective analysis and refinement processes within the DBR framework, which are often underreported in previous research.

Keywords:
APOS, Design-based research, Differential calculus, Hypothetical learning trajectory, Students' understanding

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1. INTRODUCTION

Numerous concerns about students' performance in mathematics-related subjects in the Malaysian Sijil Pelajaran Malaysia (SPM) and Sijil Tinggi Persekolahan Malaysia (STPM) examinations have emerged. Recent examination data indicated that approximately 50% of students experienced difficulties attaining a grade of C or above in Mathematics and

Additional Mathematics. One notable example was in SPM Mathematics, which the rates for those who did not score grade C and above were 52.1% in 2023, 53.3% in 2022, and 55.1% in 2021, while the rates for Additional Mathematics were 47.6% in 2023, 50% in 2022, and 48.5% in 2021 (Examinations Syndicate Ministry of Education Malaysia, 2023, 2024).

Calculus is a fundamental mathematics subject crucial for progression in more advanced mathematical fields (Ayub et al., 2005; Hanke, 2024; Portnoy, 2025). Nonetheless, mastering differential calculus presents a considerable challenge for students globally, including those in Malaysia. A significant example was observed in STPM Semester 2 Mathematics T which consists of calculus-related topics, in which the proportion of students failing to achieve a credit grade remained consistently elevated at 44.15% (2022), 42.59% (2021), and 42.63% (2020) (Malaysian Examinations Council, 2021, 2022, 2023). The Malaysian Examinations Council (2021) also documented a continued deficiency in students' comprehension of differential calculus application questions for accurately identifying stationary points in the Year 2020 Mathematics T examination report.

The Program for International Student Assessment (PISA) by the Organisation for Economic Cooperation and Development (OECD, 2019) asserted that Malaysian students encountered challenges in mathematically modelling complex situations and in choosing effective problem-solving strategies. The study underscored educators' need to prioritise enhancements in mathematics education, in which students experienced difficulties with various fundamental calculus concepts such as limits, derivatives, and rates of change. Hence, this process can adversely affect the performance of the students while reducing their motivation for pursuing advanced mathematics (Afgani et al., 2017; Borji & Martínez-Planell, 2020). The relationship between the results and applied tasks is also challenging for these students to establish. A few examples of these issues include interpreting graphical representations of functions and their derivatives or resolution of rate-of-change problems (Borgen & Manu, 2002; Ozaltun-Celik, 2021). Moreover, several studies have pinpointed that limited comprehension of the mathematical and algebraic dynamics of calculus functions has caused these challenges (Carlson et al., 2002; Haciomeroglu & Andreasen, 2013).

The Malaysian Ministry of Education and the Malaysian Qualification Agency (MQA) have acknowledged the potential of digital instruments to address these issues (Malaysian Qualifications Agency, 2019). Nevertheless, a gap persists in applying these instruments in calculus instruction, impacting students' understanding of construction (Shahriari, 2019). Despite that numerous studies have emphasised the advantages of technology in mathematics-related education (Balantes & Tonga, 2020; Dubinsky, 2014; Gertenbach & Bos, 2016), inadequate studies concerning the correlation between digital instruments such as Desmos and students' comprehension of mathematical concepts has been observed.

The traditional evaluation process of novel teaching approaches and instructional lessons has primarily focused on analysing students' performance (Evardo Jr & Itaas, 2024; Gaylo & Dales, 2017; Masinading & Gaylo, 2022; Tambaoan & Gaylo, 2019). Nonetheless, limited research examining students' comprehension of mathematical concepts has presented a significant gap. Therefore, educators should identify specific difficulties and modify their teaching techniques to effectively address these challenges by understanding how students develop their grasp of mathematical concepts. This study then proposed an instructional lesson containing Desmos and Action-Process-Object-Schema (APOS) theory to assess students' comprehension of differential calculus. The evaluation was also structured around three research questions as follows:

(i) What is the development of the students' mathematical understanding of differential calculus in application contexts from the perspective of APOS when they participate in a Desmos-based lesson?

(ii) What challenges do the students encounter when adapting to the instructional lesson designed for learning and teaching differential calculus?

(iii) What degree of alignment exists between the students' hypothetical learning trajectory (HLT) and their actual learning outcomes based on APOS?

An instructional lesson design was introduced in this study based on two theoretical frameworks: (i) APOS and (ii) HLT. These frameworks offered a theoretically grounded methodology for examining the students' comprehension of differential calculus concepts by guiding the lesson design. Each theory provided distinct insights into student cognition, the organisation of learning activities, and the anticipation of student responses. The following subsections inform the instructional design and the analysis of student understanding by thoroughly examining the theoretical frameworks in this study.

1.1. APOS Theory and Students' Understanding in Calculus

Dubinsky (1991) pioneered the APOS theory as a constructivist framework for assessing students' mathematical understanding based on four phases: (i) Action, (ii) Process, (iii) Object, and (iv) Schema. This theory posits that the initial construction of mathematical knowledge is at the Action phase, in which external guidance and sequential routine processes influence the students' comprehension (Asiala et al., 1997). The Process phase is then attained when the students advance in proficiency, and they internalise these actions and can execute them autonomously. Subsequently, students view processes as complete entities in the Object phase, acknowledging the entirety of a concept. These students can finally apply their knowledge flexibly in the Schema phase, representing a structured conceptual framework that integrates Action, Process, and Object (Dubinsky & McDonald, 2001).

The APOS theory in higher education usually involves investigating the students' mental constructions and their comprehension of mathematical concepts by educators using technology (Arnawa et al., 2020). A theoretical framework is also presented within the genetic decomposition of this theory for forecasting the students' cognitive constructions during the learning process (Arnon et al., 2014). Moru (2020) examined the students' comprehension regarding the association between secant and tangent lines in derivatives using the APOS framework. Even though the study indicated that the students attained a Process phase of understanding algebraic tasks, they encountered difficulties with graphical tasks. Borji et al. (2018) denoted enhanced graphical comprehension in the students utilising Maple software compared to those receiving traditional lecture-based instruction. Arnon et al. (2014) presented improved Process concepts of functions and understanding of derivatives when activities-classroom discussion-exercise (ACE) cycles and computer-assisted activities were employed.

Previous studies suggested several strategies that educators could apply in their teaching lessons. One prominent example was employing the initial APOS genetic decompositions to develop instructional activities containing interactive web-based tools while assessing the effectiveness with students (Burns-Childers & Vidakovic, 2018). These studies then successfully indicated the importance of technology in calculus instruction and its effects on achievement and attitudes (Albalawi, 2018; Ebert, 2014). Nonetheless, insufficient APOS-related studies concerning students' understanding of newly designed technology-enhanced environments with Desmos were observed. Thus, this study assessed the students' cognitive development during their engagement with Desmos activities-based APOS to examine their progression and challenges in comprehending derivatives.

1.2. Hypothetical Learning Trajectory (HLT)

Educators can anticipate students' cognitive processes during instruction by applying hypothetical learning trajectory (HLT) as a theoretical framework to forecast students' learning pathways. This framework generally comprises three primary elements: (i) learning goals, (ii) hypothetical students' learning, and (iii) alignment between expected and actual learning outcomes (Simon, 1995). Bakker (2004) argued that instructional theory and practical classroom application could be connected using HLT. Morales Carballo et al. (2022) explained that educators could produce activities that achieved these outcomes by establishing explicit learning objectives within an HLT. Hendrik et al. (2020) stated that educators could enhance intervention to support students' learning and address conceptual difficulties by assessing the alignment between lessons and initial planning.

Numerous studies successfully employed HLT for differential calculus instructions. Khairudin et al. (2022) established dynamic classroom environments that improved student engagement while examining the students' conceptual understanding using HLT. Yarman et al. (2020) assessed students' mathematical reasoning regarding population growth within modelling approaches based on HLT. Consequently, a comprehensive framework offering predictive insights for instructional design and assessment was accomplished by integrating APOS and HLT.

The HLT typically delineates anticipated learning outcomes, whereas APOS presents information about the cognitive processes underlying students' comprehension. Therefore, this study demonstrated how the integration of these frameworks influenced an instructional design containing Desmos activities to produce a theoretically grounded and adaptable approach for teaching differential calculus. This dual-framework approach facilitated adaptable and responsive instruction corresponding to the student's cognitive pathways while offering a systematic strategy for examining student learning.

2. METHOD

This study utilised a design-based research (DBR) strategy for instructional design consisting of three stages: (i) preparation and design, (ii) teaching experiment, and (iii) retrospective analysis and refinement. The DBR strategy also improved the intervention and the desired learning outcomes by facilitating the design, implementation, observation, and refinement of the intervention in real time. Initially, students' weaknesses in a specific subtopic of differential calculus were collected using a diagnostic test in the preparation and design stage. Three mathematics education experts verified this diagnostic test guiding the instructional lesson design to confirm that the questions accurately examined students' foundational understanding of differential calculus. Subsequently, Desmos produced the intervention, which was an instructional lesson in differential calculus. The APOS and HLT frameworks also guided this intervention.

The second stage involved conducting the teaching experiment following the complete design and validation of the instructional lesson. This teaching experiment was performed with a foundational science class in a pre-university programme at a private university in Sarawak, Malaysia. Data was then gathered to address the research questions based on the students' work from the Desmos platform, classroom observations, and student interviews. An experienced course coordinator serving as a teaching witness also verified the lesson content. This witness assessed the suitability of the materials employed in teaching differential calculus by evaluating each session while providing feedback through an assessment form to determine the effectiveness of the lesson.

Individual interviews were conducted with nine students purposefully selected based on their diagnostic test performance to ensure a range of academic abilities was represented. Three students were chosen from each performance category: (i) above-average (S1, S2, S3), (ii) average (S4, S5, S6), and (iii) below-average (S7, S8, S9). According to Patton (2002), this stratified selection aimed to capture diverse perspectives and experiences across different achievement levels, thus enhancing the depth and credibility of findings and also providing a more comprehensive understanding of student learning.

Semi-structured interviews were employed to encourage open-ended responses, allowing participants to share their thoughts in depth. As Creswell (2012) noted, interviews can enhance researchers' understanding of participants' perspectives. In this study, the interviews specifically focused on gathering student feedback regarding the Desmos-based instructional lesson.

The interviews were recorded, transcribed, and analysed qualitatively using ATLAS.ti 23 software. Thematic analysis was conducted following the six-phase process described by Braun and Clarke (2006), which includes data familiarisation, initial coding, theme identification, theme refinement, theme definition, and final reporting.

To ensure the credibility of the findings, the "member checking" approach (McKim, 2023; Thomas, 2017) was employed, where participants reviewed and verified their transcripts. Consistent with the recommendations of Bogdan and Biklen (1997), this iterative validation process helped strengthen the reliability and trustworthiness of the qualitative data.

The third stage of DBR involved a retrospective analysis to assess the alignment between students' actual learning outcomes and the expected learning trajectories. This analysis was conducted through a systematic review of the classroom observation and students' work on Desmos platform.

A symbolic coding system was used to represent the degree of alignment:

(i) a "+" was assigned when students' responses and behaviors demonstrated strong evidence of achieving the intended learning outcomes,

(ii) a "0" indicated partial or ambiguous alignment, where students showed some understanding but with notable gaps or inconsistencies, and

(iii) a "-" was used when students' responses clearly deviated from the expected trajectory, showing misconceptions or lack of engagement with the core concepts.

Based on this coding analysis, areas of misalignment were identified, and the instructional lesson was refined to improve its effectiveness in subsequent iterations.

2.1. Participants

This study involved 32 pre-university students, aged 17 to 19 from a foundation in science programme at a private higher education institution in Sarawak, Malaysia. Participants were chosen using a combination of convenience and purposive sampling to capture a spectrum of calculus proficiency levels. A diagnostic test results was administered to classify students into three proficiency categories: (i) above-average, (ii) average, and (iii) below-average. For the teaching experiment sessions, a focused sample of nine students was selected, which was three from each category. This approach aligned with design-based research methodology, which emphasizes in-depth exploration of learning processes over broad generalization (Barab & Squire, 2004; The Design-Based Research Collective, 2003). Purposeful sampling in small groups allows for rich and context sensitive insights into the design and refinement of educational interventions (Cobb et al., 2003; Lakens, 2022). The selected nine students were closely observed during the teaching experiment sessions, and their work provided valuable data on learning trajectories, misconceptions, and areas for instructional improvement.

2.2. Data Collection

The data collection process involved information from various sources. A diagnostic test was initially administered to assess the student's comprehension of differential calculus concepts while pinpointing areas of weakness before the intervention. Significant data was then recorded from the students' engagement with the Desmos platform. This platform containing the students' interactions provided two primary information concerning the lesson: (i) comprehension and (ii) misconceptions of the students. An APOS theory framework was finally employed to examine the students' comprehension.

Observational video recordings were conducted during the lesson. This study initially documented the session in real-time to analyse the students' learning environment during the teaching experiment stage. An HLT table was then applied to perform the retrospective analysis involving the systematic documentation of the student's actual learning progress and its comparison with the anticipated learning outcomes. Interviews were finally conducted with nine selected students following the lesson. These semi-structured interviews also facilitated the student discussions concerning their learning experiences and feedback on the Desmos activities.

3. RESULTS AND DISCUSSION

3.1. Results

3.1.1. Data Analysis from Stage 1: Design and Preparation

The initial data analysis stage examined students' diagnostic test results to inform instructional content development tailored to their needs. Therefore, the lesson and HLT were customised to address these needs effectively following identifying weaknesses in differential calculus.

Analysis of Diagnostic Test Results

Table 1 tabulates the diagnostic test results administered to 32 participants based on five questions. Question 1 in the diagnostic test required students to identify the x-coordinates of the stationary point for a specified equation. Question 2 evaluated the students' understanding of normal and tangent lines. Question 3 necessitated the students to analyse information from presented graphs and sketch the graph of the derivative based on their interpretation. Question 4 presented an optimisation problem, requiring the students to determine the maximum volume of water that a tank could accommodate. Question 5 asked the students to construct a cubic graph based on a given equation. Consequently, the findings demonstrated that the students encountered the most significant difficulties with Questions 4 and 5, which required optimisation and graphing skills. This outcome implied that particular emphasis was placed on these areas in designing the instructional lesson, focusing on optimisation to address this need.

Table 1. Summary of the students' diagnostic test results

Percentage of Students in APOS Level (%)	Question Number				
	1	2	3	4	5
Zero marks/no response	15.6	25	21.9	34.4	43.7
Action	25	6.3	21.9	50	40.7
Process	34.3	40.6	28.1	15.6	12.5
Object	18.8	6.3	28.1	0	0
Schema	6.3	21.8	0	0	3.1

Design of HLT

This study designed the lesson to incorporate the Desmos platform upon finalising the instructional lesson content. Table 2 lists the design of HLT where the lesson is accomplished entirely through Desmos.

Table 2. Summary of the design of HLT

Activity	Conjecture Regarding the Conduct of the Lesson	Evaluation of Students' Understanding Based on APOS Theory
Introduction and Whole-Class Discussion	Students could articulate and analyse the similarities and differences in the derivatives of two cubic functions.	Action: Students could recognise that both derivative graphs represent quadratic functions.

Activity	Conjecture Regarding the Conduct of the Lesson	Evaluation of Students' Understanding Based on APOS Theory
		Process: Students could articulate the correlation between the increasing or decreasing behaviour of graph f and the positive or negative value of graph f' across the relevant intervals. Object: Students could compare the graphs of the two derivatives Schema: Students could establish connections among Action, Process, and Object (slope, tangent line, and stationary point) to construct a Schema for comparing the derivative functions of the two graphs involving f.
Lesson Content - Optimisation Problems	The instructor illustrated the four-step method for solving optimisation problems. Students could create a diagram representing the overall problem, derive equations from the word problems, and solve them using the first derivative and sign diagram techniques following the lesson.	Action: Students were tasked with creating a clear diagram derived from the information in the word problem statements. Process: Students could identify the conditions necessitating optimisation and structuring the word problem into equation format. Object: Understanding the object allowed students to execute operations on the formulated equation, offering a systematic approach. Schema: Students could produce a Schema (sign diagram or other constraints) to examine the optimisation problem by connecting Action, Process, and Object.
Group Activity	Students could independently explore a new optimisation problem upon completing the optimisation problems without instructor guidance while being capable of collaborating effectively in groups. Students could generate diagrams and formulate mathematical equations based on word problems. These students could also address optimisation problems through the first derivative and sign diagram technique.	Action: Students could construct a precise diagram derived from the information presented in the word problem statements. Process: Students could determine the conditions necessitating optimisation while translating word problems into equation format. Object: Comprehending the Object enabled students to execute operations on the formulated equation while offering a systematic approach.

Activity	Conjecture Regarding the Conduct of the Lesson	Evaluation of Students' Understanding Based on APOS Theory
		Schema: Students could develop a Schema (sign diagram or relevant constraints) to assess the optimisation problem by connecting Action, Process, and Object.
Take Home Exercise	The instructor provided a briefing and shared the Desmos link with students, enabling them to complete the task independently at home.	N/A

Note: N/A = Not available.

3.1.2. Data Analysis from Stage 2: Teaching Experiment

Classroom Observations During the Teaching Experiment

Figure 1 depicts the actual classroom environment during the teaching experiment stage. The lesson occurred in a computer lab to facilitate the use of desktop computers, which were necessary for employing the Desmos platform during the session. Cameras were also installed in the classroom to document the overall environment while concentrating on three designated student groups, each comprising three students from varying achievement levels.

Figure 1. The classroom environment overview for the teaching experiment

The instructor initiated the lesson with an introductory whole-class session, utilising a cubic graph on the Desmos platform to evaluate students' comprehension of its derivative properties. Subsequently, the students were assigned to create a sign diagram and sketch the derivative graph. The knowledge acquired in secondary school also served as a prerequisite for addressing derivative application problems. These student responses were presented on the projector screen through the instructor's Desmos interface. Consequently, the responses indicated that certain students encountered difficulties with the sign diagram in accurately labelling the x-coordinate. The x-coordinate of the stationary point was incorrectly labelled as the x-intercept of the original graph, reflecting a misunderstanding of the x-coordinate location concerning the turning point. Figure 2 portrays the errors committed by students in sketching the sign diagram for a cubic function.

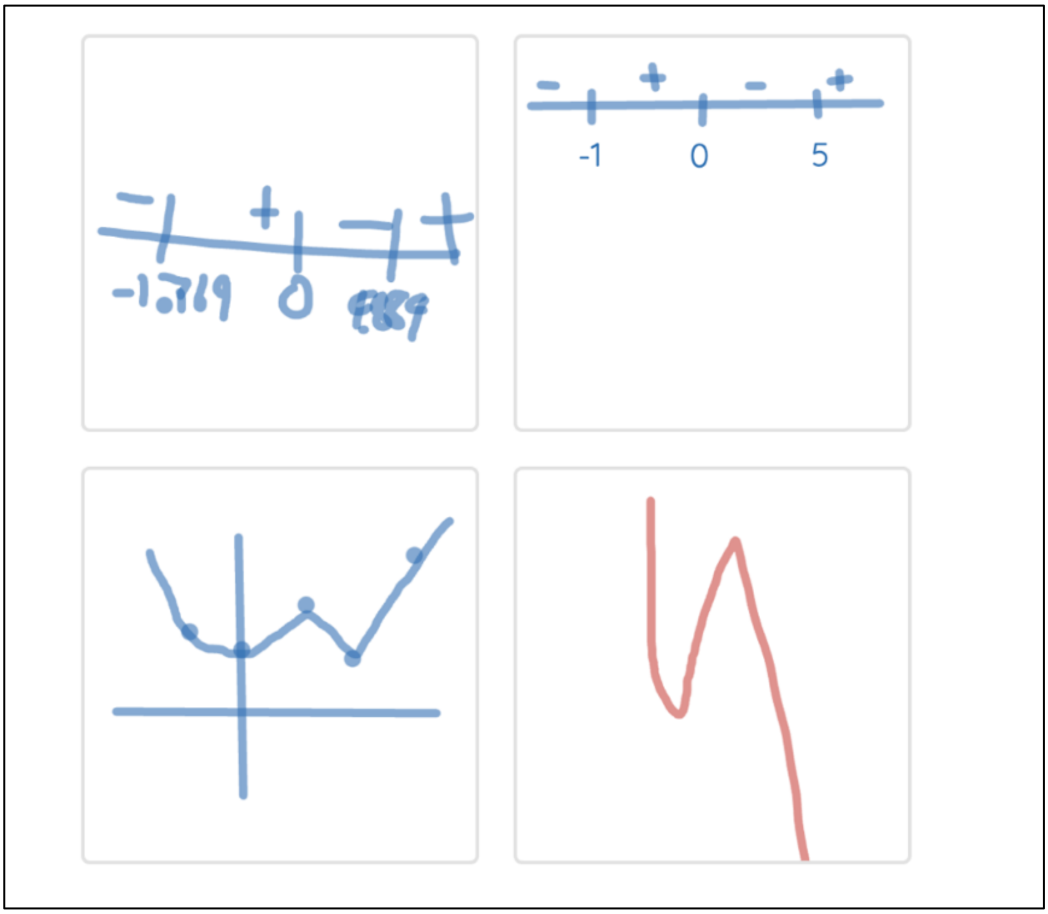

Figure 2. Several examples of errors made by students in the sign diagram task

The students identified that the derivative graphs exhibited a quadratic shape in the class discussion concerning the similarities and differences in the derivative graphs of two cubic functions. Two groups observed that the derivative graphs exhibited identical *x*-intercepts. The remaining students needed assistance from the instructor to comprehend that this characteristic stems from the original graphs possessing stationary points at the same x-values. These students also described the two distinct shapes of the graphs (u-shaped and n-shaped) through a sign diagram.

The instructor presented a four-step method for addressing word problems in the context of optimisation during the main lesson content. This process involved four sequential steps: (i) creating a precise diagram to illustrate all essential components, (ii) formulating an equation with the variable to be optimised, (iii) computing the first derivative while identifying the x-values at which it equalled zero, and (iv) employing a sign diagram to ascertain whether the stationary point indicated a maximum or minimum. The instructor also guided the students through an example optimisation problem utilising these steps.

Although the students collaborated on tasks during the group activity, some opted for independent work. Subsequently, a student was chosen to present the solution of the group to the class. This student presented the initial two steps involving creating a diagram and formulating an equation derived from the specified problem. Nonetheless, this student could not advance to the final two steps due to time limitations. Figure 3 presents the solutions analysed by the group of this student. The instructor was also unable to complete all planned tasks and the final task concerning the area for optimisation problems. Nevertheless, the instructor reviewed the four primary steps for solving optimisation problems to reinforce the key concepts from the lesson.

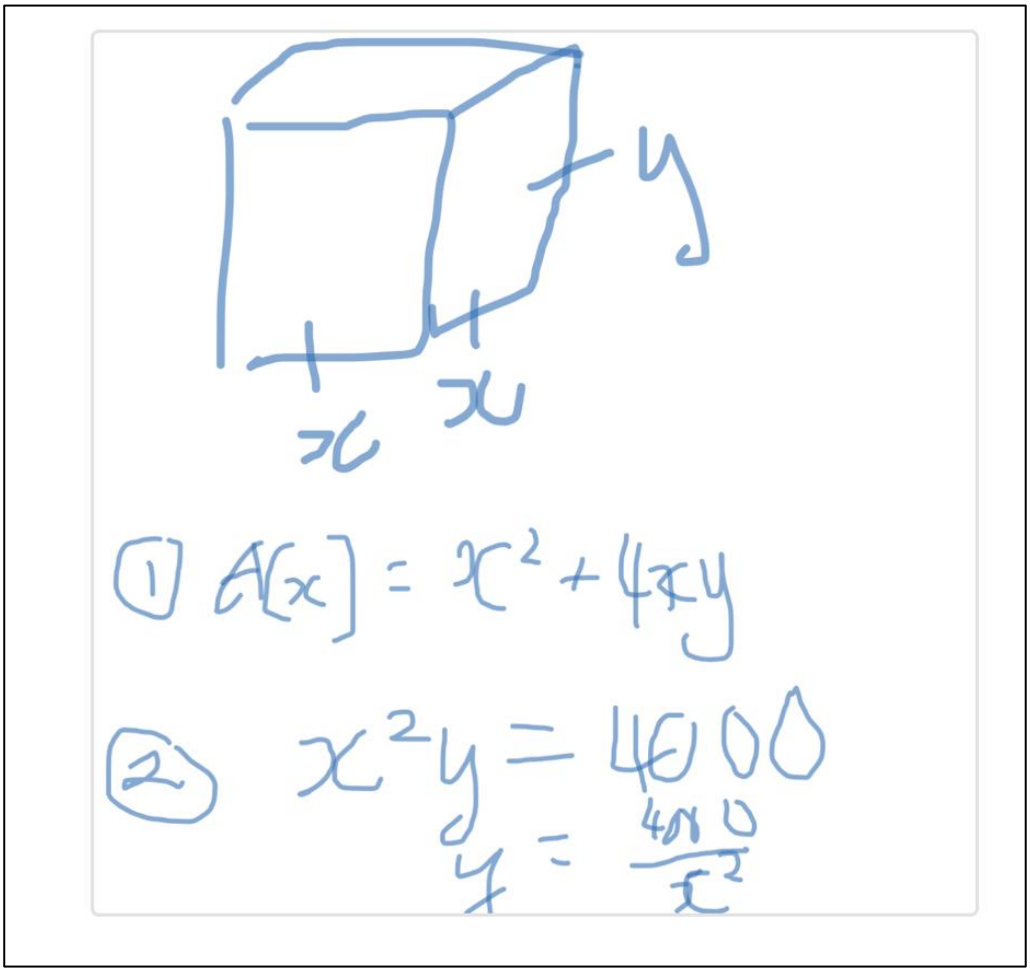

Figure 3. An example of a collaborative student work

Students' Feedback on the Newly Designed Instructional Lesson

The individual semi-structured interviews were employed to elicit open-ended responses from participants. Two primary themes emerged from the student interviews in this study: (i) the advantages of the instructional lesson and (ii) the challenges students encountered in adapting to the designed instructional lesson for learning and teaching differential calculus.

(1) Theme 1: Benefit of the Lesson

Interviews with students of varying proficiency levels in calculus revealed several key sub-themes related to the benefits of the lesson: (a) interesting lesson, (b) enhanced understanding, and (c) opportunities for group discussion.

a. Interesting Lesson

The lesson utilising Desmos was deemed interesting and enjoyable by the students across various proficiency levels. One significant example involved two above-average students (S1 and S2) and one below-average student (S8), which indicated that Desmos enhanced their enjoyment of differential calculus classes and heightened their interest in the subject matter. Several students also implied that their initial experience utilising Desmos in their calculus studies contributed a sense of novelty to their learning process.

Instructor : What do you think about Demos?
S1 : I think it is interesting.

(Excellent_S1_29-30)

Instructor : Have you previously used technology tools like Desmos in your derivative calculus class?
S2 : No, this is the first time.
Instructor : So, this is your first time, right? Can you tell me one factor you found helpful in learning mathematics, especially on differentiation?
S2 : Using Desmos makes it a bit more fun. I can plot my graphs.

(Excellent_S2_37-40)

Instructor : Did you like how the instructor taught the differentiation topic using the Desmos platform?
S11 : Yes.
Instructor : Why?
S11 : It's interesting.

(Poor_S11_99-102)

b. Promoting Understanding

The use of Desmos significantly promoted students' understanding. S2, S3, and S5 appreciated that Desmos recognised that Desmos enabled the anonymous display of their answers, facilitating immediate peer comparison and enhancing their learning experience. S7 also observed that viewing classmates' responses on Desmos promoted learning from diverse approaches and improved their comprehension of mathematical concepts.

Instructor : How does it help you understand better?
S2 : We can type out answers, see our classmates' answers, and learn from classmates' answers displayed.

(Excellent_S2_47-48)

Instructor : Do you like the way the instructor uses technology in class?
S9 : Yes. It's more comfortable. I can even refer to classmates' answers. I understand each other's mistakes and learn better, even better.
(Poor_S9_59-60)

c. Opportunities for Group Discussion

The lesson effectively lesson promoted collaborative group interactions among students. Notably, S3 and S5 appreciated the opportunities for group discussion when addressing challenging questions.

Instructor : ... which part of our lesson process is most helpful for understanding the topic?
S5 : The group discussion session.
Instructor : Why is that?
S5 : Because we can discuss answers and understand better.
(Medium_S5_58-61)

(2) Theme 2: Students' Obstacles

Students frequently reported challenges in typing equations on the Desmos platform. One considerable example was S7 articulating frustration regarding the input of mathematical equations and the search for specific symbols. On the contrary, S8 proposed enhancements to improve usability in the Desmos application.

Instructor : Are there any difficulties you faced using Desmos?
S7 : Sometimes, finding the symbols and typing the equations is hard.
(Poor_S7_107-109)

Instructor : Any difficulties with operating the platform?
S8 : Editing formulas.
Instructor : Editing the formulas?
S8 : Yes.
Instructor : Like typing math equations?
S8 : Yes, sometimes I struggle with finding the fraction button.
Instructor : Okay. Can you suggest how to improve this platform's use in lessons?
S8 : Maybe add ready-made equations for easier access.
(Poor_S8_127-132; 137-138)

3.1.3. Data Analysis from Stage 3: Retrospective Analysis and Refinement

The retrospective analysis and refinement are two key elements on Stage 3 in design-based research.

Retrospective Analysis on HLT Table

The retrospective analysis in Stage 3 of DBR involved comparing students' actual learning with the expected outcomes outlined in the HLT table. This analysis provided data on aligning actual learning with the hypothetical trajectory. Consequently, educators could adjust and refine the designed instructional lesson based on this comparison (see Table 3).

Table 3. Comparison summary of the HLT and ALT results

Hypothetical Learning Trajectory (HLT)		**Actual Learning Trajectory (ALT)**	
Task Formulation	**Conjecture regarding the conduct of the lesson**	**Transcript excerpt or observation obtained**	**Quantitative match between HLT and ALT (+, 0, -)**
Introduction and Whole-Class Discussion	The students could articulate and analyse the similarities and differences in the derivatives of two cubic functions.	The students recognised that the derivative graphs exhibited a quadratic form. Two groups identified that the derivative graphs intersected the x-axis at identical points. Given that the original functions exhibited the same x-coordinates for their stationary points, certain students required assistance from the instructor to understand that the derivative graphs possessed identical x-intercepts. Only one student noted that the derivative graphs denoted an equal number of turning points. The students accurately observed that the graphs exhibited distinct shapes. These students' discussions also included differences in the sign diagrams, noting that one graph was u-shaped while the other was n-shaped.	+
Group Activity	The students could independently analyse a new optimisation problem without instructor guidance and collaborate effectively in groups. The students could create diagrams and	The students completed the initial two steps: (i) producing a diagram based on the word problem and (ii) formulating an equation following the specified conditions. Considering that the volume was delineated in the problem, a student representative accurately	-

Hypothetical Learning Trajectory (HLT)		**Actual Learning Trajectory (ALT)**	
Task Formulation	**Conjecture regarding the conduct of the lesson**	**Transcript excerpt or observation obtained**	**Quantitative match between HLT and ALT (+, 0, -)**
	formulate mathematical equations based on word problems. These students also addressed optimisation problems by applying the first derivative and sign diagram technique.	articulated the area function with vertical sides in the variable x. The instructor discontinued the group activity due to time constraints. Consequently, the lesson ended with the instructor summarising the four steps for solving optimisation problems to reinforce the key concepts discussed.	
Take Home Exercise	The instructor provided a briefing and shared the Desmos link with students, enabling them to complete the task independently at home.	N/A	0

Retrospective Analysis of Students' Work

This study retrospectively analysed the students' work stored on the Desmos platform. The storage enabled the instructor to assess their work post-lesson, which the activity involved students comparing the similarities and differences between the derivatives of two specified cubic graphs during the introduction and whole-class discussion. Figure 4 displays the students' responses on the Desmos platform. The findings revealed that nearly all groups (except one group) accurately recognised the derivative functions as quadratic. This outcome suggested that most students had achieved at least the Process phase.

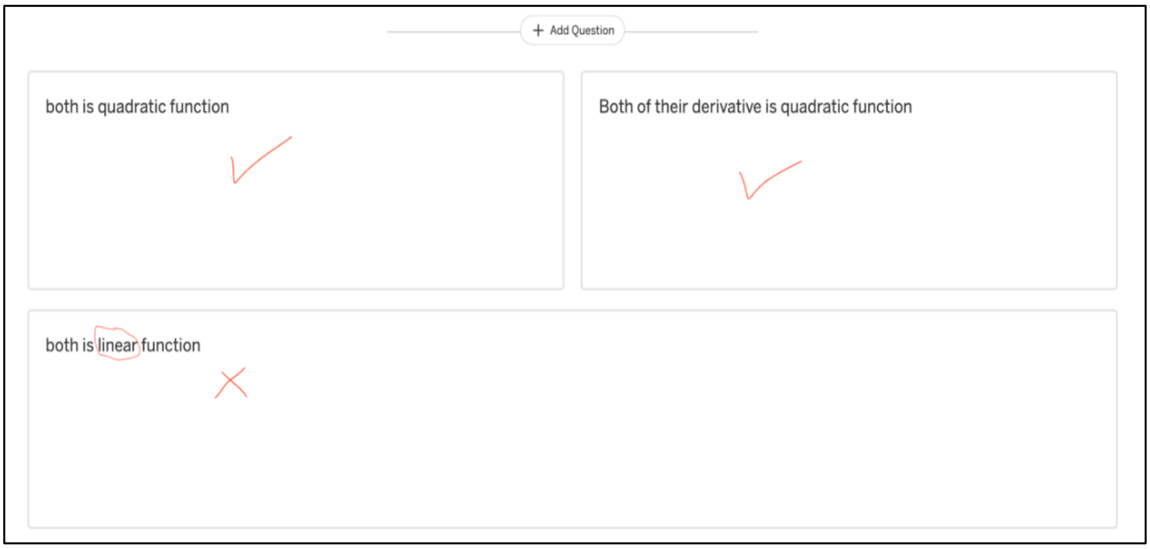

Figure 4. Example of students' answers in introduction and whole-class discussion activity (Part 1)

Figure 5 depicts that two groups further establish that both derivative graphs intersect the x-axis at identical points. This observation signified that the groups had attained the Object phase. Nevertheless, none of the groups developed the Schema phase or discussed the derivative functions more deeply.

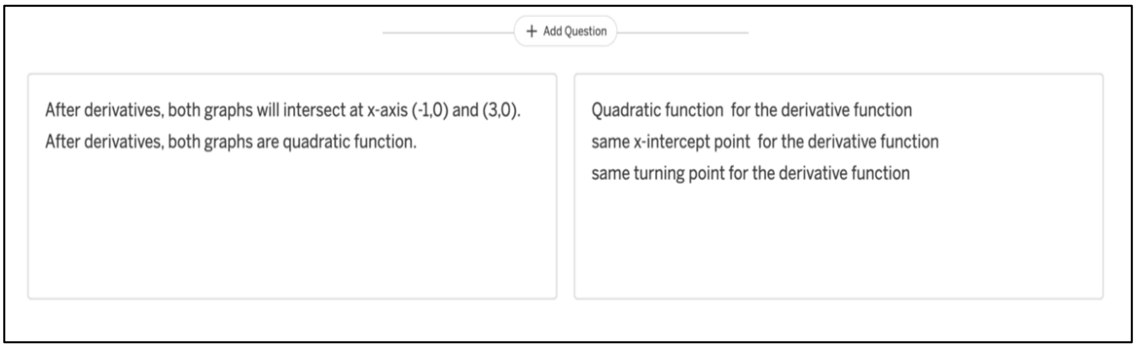

Figure 5. Example of students' answers in introduction and whole-class discussion activity (Part 2)

Despite specific diagrams being inadequately labelled, all groups also effectively produced diagrams based on the word problem in the group activity focused on solving a word problem related to optimisation for minimising the surface area of a box with a square base (see Figure 6).

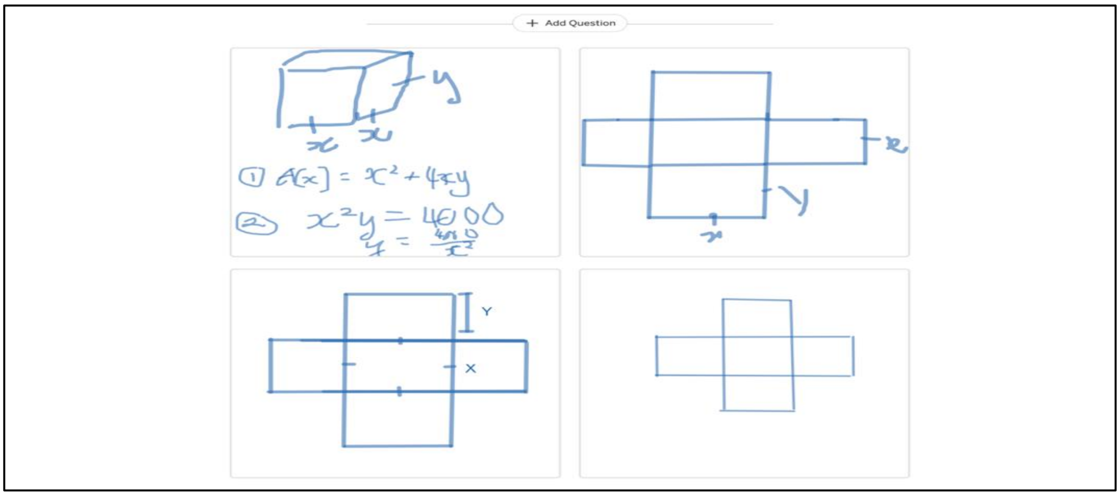

Figure 6. Example of students' answers in drawing diagrams for optimising word problem

Figure 7 illustrates the students' proficiency in the Process phase by accurately formulating the area equation in a single variable. Only two groups achieved the Object phase by deriving an equation to minimise the surface area (see Figure 8). The remaining groups failed to obtain the correct derivative equation (see Figure 9). Consequently, the analysis indicated that most students were in the Process phase based on the observed teaching episodes.

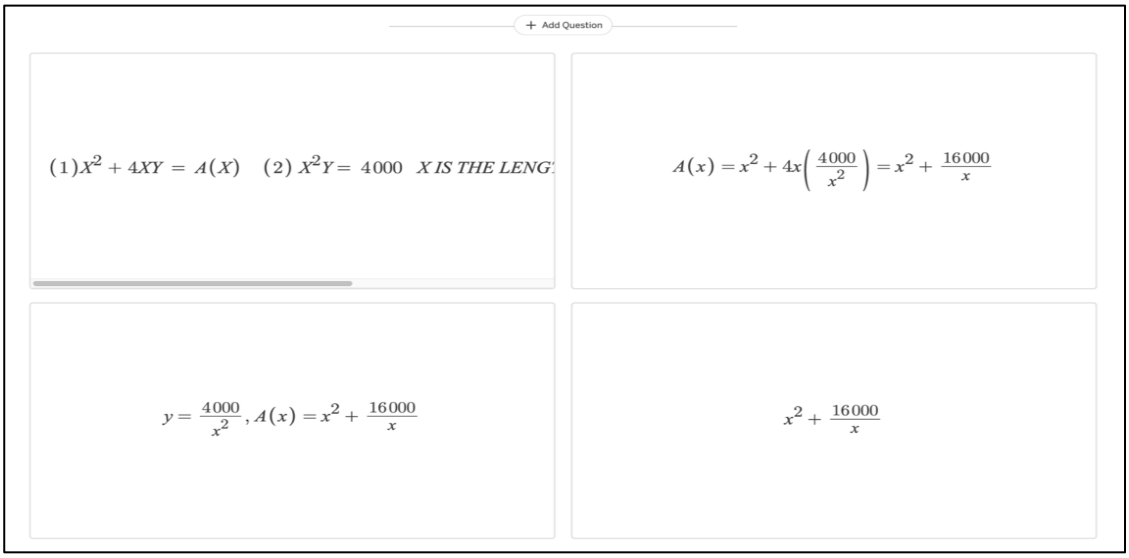

Figure 7. Example of students' answers in formulating equations for the optimisation problem

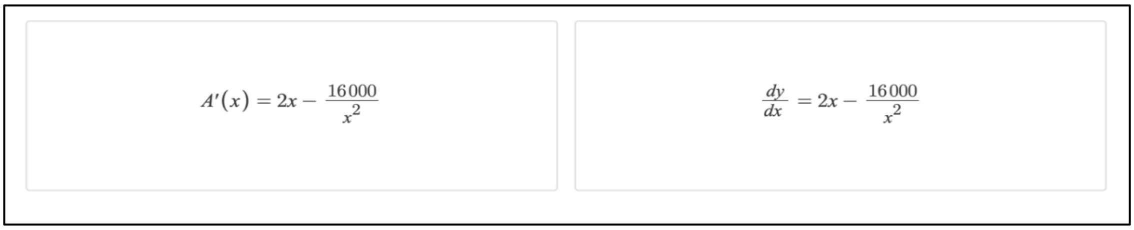

Figure 8. The correct answers obtained from two groups for the derivative equation

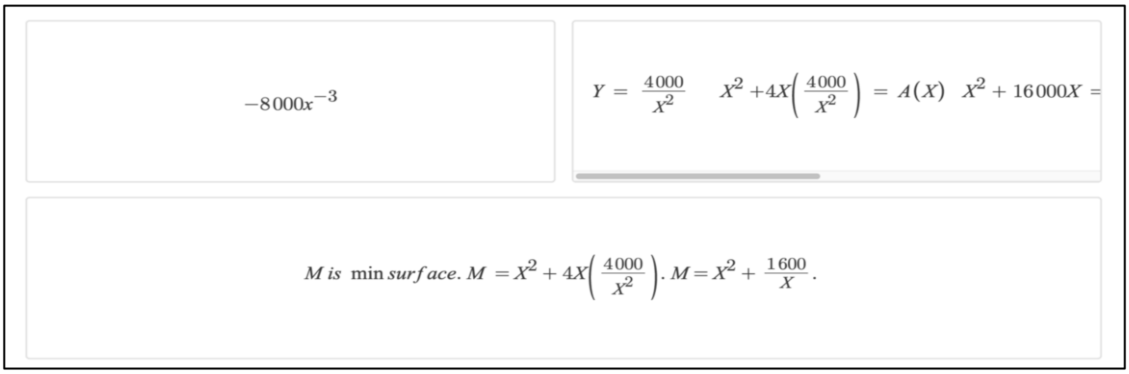

Figure 9. The incorrect answers from the remaining groups for the derivative equation

Refinement of the Designed Instructional Lesson

Two refinements to the instructional lesson were accomplished based on the student feedback, classroom observations, and retrospective analysis. The retrospective analysis of HLT initially demonstrated that most components were denoted with a "-". This outcome indicated a misalignment between actual learning outcomes and the intended learning objectives, which was attributed to the length of the lesson content impeding the timely completion of group discussions and presentations. Therefore, this study revised the introduction to concentrate exclusively on individual work, directing students to articulate their understanding on the Desmos platform rather than engaging in whole-class discussion. This modification could minimise the time spent reviewing existing knowledge regarding graph properties.

Given that many students lacked familiarity with inputting mathematical symbols, the student feedback revealed a prevalent challenge in entering equations on the Desmos platform. Thus, this study implemented alternative assessment activities (matching graphs or selecting correct statements through checkboxes) to evaluate their understanding rather than requiring students to type solutions. This process would conserve time while enabling comprehension evaluation through various activity types available on the Desmos platform.

3.2. Discussion

This study successfully demonstrated that DBR in instructional lesson design ensure two primary outcomes: (i) emphasis on the design and (ii) the necessity of testing and adapting lessons based on students' understanding and feedback (Alias & Hashim, 2012). The findings indicated that most students fully comprehend derivatives, achieving only the Process phase. This outcome was consistent with the outcomes of Haghjoo and Reyhani (2021) who also observed that students generally demonstrated a weak understanding of derivative concepts, with the rate of change topic being the most challenging area.

Even though the students could construct accurate diagrams and formulate equations from word problems, these students encountered difficulties in justifying value rejections and making conjectures for optimisation problems. This observation suggests that most students were operating at the Process phase, indicating a gap in their understanding of differential calculus, as they had not yet fully transitioned to the Object phase, where conceptual knowledge becomes more abstract and generalisable. A significant number of students lacked the cognitive frameworks required for advanced thinking. Feudel and Biehler (2021, 2022) observed that students frequently misinterpreted derivatives in economic contexts. Consequently, this study indicated that significant information for enhancing instructional strategies could be realised by employing APOS theory in document analysis to analyse students' graphical comprehension of derivatives.

The student interviews in this study revealed that participation in group activities on the Desmos platform enhanced lesson engagement. Specifically, the instructional lessons designed utilising Desmos provided a significant advantage by facilitating learning. This process enabled the students to compare solutions with peers through the teacher dashboard, improving the learning process while deepening the understanding of mathematical concepts. The outcomes of this observation also aligned with the findings of TLS and Herman (2020), and Liang (2016). These studies documented that visualisation production, idea sharing, and effective engagement with educators could be attained through the graphing feature of the Desmos platform.

Integrating Desmos in the classroom enabled educators to assess its influence on student engagement and comprehension more effectively (Alvarez & Galman, 2024; Bastos, 2022; Chechan et al., 2023; Meto & Paleta, 2025). This study effectively projected the students' work on the classroom screen to address misconceptions and facilitate student understanding. King (2017) published that increased student engagement and conceptual understanding of various functions could be accomplished using Desmos. Cambrian College Teaching & Learning Innovation Hub (2024) and Gulati (2016) asserted that a visually engaging and interactive student experience was observed when Desmos was applied to convert abstract mathematical concepts.

While Desmos significantly enhanced student engagement and comprehension, a notable difficulty arose was students' struggles with the platform's interface, particularly in inputting equations and interpreting the visual outputs. These challenges were especially facing by students in the Action phase of APOS (Listiawati et al., 2025), who were still focused on basic interactions with the technology rather than engaging with the underlying mathematical concepts. As highlighted by Hillman (2014), the complexity of using technology can hinder students' ability to effectively transition into more abstract thinking. To address these issues, future lessons may need to offer more guided support in navigating Desmos.

The alignment between the hypothetical learning trajectory (HLT) and the actual learning trajectory (ALT) was evaluated through classroom observations and retrospective analysis. While most students reached the Process phase in their understanding, there were discrepancies in achieving the higher-order cognitive goals predicted in the HLT. These findings indicate that while the HLT provided a strong framework for the expected learning progress, further refinement of the instructional design is necessary to bridge the gap between HLT and ALT, particularly in encouraging deeper conceptual understanding (Bakker & van Eerde, 2015; McKenney & Reeves, 2019).

The refinement stage of DBR incorporated student feedback on the newly designed instructional lesson. This feedback was crucial to enable instructors or researchers to refine the teaching approaches while adapting content to meet students' requirements better. Student feedback served as a crucial data source for evaluating the alignment between the hypothetical learning trajectory (HLT) and the actual learning trajectory (ALT). In design-based research, student reflections and responses provide direct insight into their conceptual understanding, misconceptions, and perceived learning experiences, where these elements are central to assessing whether the instructional design met its intended goals (Bakker & van Eerde, 2015; McKenney & Reeves, 2019). Mandouit (2018) discussed that student feedback was crucial to providing a framework for identifying areas needing improvement in teaching practices and aiding teachers in comprehending student challenges in the classroom. This study also noted that students encountered challenges with specific features of Desmos when operating the platform. Hillman (2014) determined that students' efficiency could be impeded if complexity was high in using technology in the classroom. Thus, the simplification of Desmos in this study for equation input could facilitate students' transition from traditional classroom settings to technology-oriented environments (Chorney, 2022).

4. CONCLUSION

Recent studies have highlighted the significance of technology in enhancing student performance. Numerous studies on technology-integrated instructional design have predominantly concentrated on evaluating instructional materials, frequently neglecting students' comprehension and the practical difficulties encountered in actual classroom environments. Therefore, this study successfully enhanced pedagogical theories by incorporating established APOS theory and hypothetical learning trajectory (HLT) to design, implement, and revise instructional lessons. This study also aligned instructional design with APOS theory, yielding insights into the stages of student learning in differential calculus while enhancing educators' understanding of students' cognitive processes in mathematics. Consequently, the outcomes of this study offered targeted analyses relevant to each stage of DBR to improve existing literature. This study also methodologically illustrated the execution of teaching experiments while highlighting the role of qualitative data such as classroom observations, student work on Desmos, and interviews in enriching the teaching experiment stage and instructional lesson design. Nonetheless, future studies should include additional statistical analyses to deepen the understanding of how instructional design impacts student learning outcomes. While student feedback alone cannot fully confirm that the HLT and ALT were perfectly aligned, it can be triangulated with insights from teaching witnesses during the teaching experiment sessions in the future study, as well as with students' observed performance and task completion. This triangulation strengthens our confidence that the gap between the hypothetical and actual learning trajectories was meaningfully reduced.

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