Infinity Journal of Mathematics Education
Volume 15, No. 2, 2026

p-ISSN 2089-6867
e–ISSN 2460-9285

https://doi.org/10.22460/infinity.v15i2.p529-544

Types of mathematical difficulties experienced by secondary school students in problem solving

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Received: Nov 17, 2025 | **Revised**: Mar 26, 2026 | **Accepted**: Apr 1, 2026 | **Published Online**: Apr 28, 2026

Abstract

Throughout the history of scientific knowledge development, we can find a wide variety of difficulties that, although potentially error-prone, have contributed to the advancement of science. This study was conducted with the aim of identifying pupils' performance and the difficulties in solving types of mathematical problems contextualized to the quadratic function and constitutes a descriptive-quantitative study. The data were obtained from 304 secondary school students in two regions of Chile, using a problem-solving test. According to the results, the greatest difficulties were found in the complexity of mathematical objects and mathematical thinking processes, surpassing the difficulties associated with teaching processes with which they were compared. With regard to the types of problems defined according to their nature and context, the lowest performance was obtained in non-routine problems and the highest was found in routine problems and those of a fantasist and purely mathematical nature. The results provide information for future research and encourage changes in school practice, which should take into account overcoming difficulties by eliminating them or by exploring their potential.

Keywords:
Assessment, Mathematical difficulties, Problem solving, Quadratic function, Secondary school students

How to Cite:
Díaz, V. (2026). Types of mathematical difficulties experienced by secondary school students in problem solving. *Infinity Journal*, *15*(2), 529-544. https://doi.org/10.22460/infinity.v15i2.p529-544

1. INTRODUCTION

In the current context of globalisation, although teaching tends to develop problem-solving skills, particularly under criteria that arise in various contexts close to reality, in real life, students are not faced with an equation or a function, but rather with a problematic situation in everyday life to which they must find some kind of solution, part of which involves using a mathematical tool that serves as an instrument for finding an answer to the problem posed. A situation is called a problem when there is an awareness of the importance of taking action, but it cannot be done immediately. If the student is ready to provide a solution strategy for a mathematical problem, then the question is no longer a problem, but an exercise

(Schoenfeld, 2013). It is characteristic of a problem that there is a difficulty that requires a solution that invites reflection (Olivares et al., 2020).

This is associated with the usefulness of incorporating all types of problems, such as routine and non-routine context problems (Csapó & Funke, 2017; Díaz et al., 2025; Kholid et al., 2024), but unfortunately, teachers are reluctant to routinely implement such tasks (Cheeseman et al., 2013; Sukarma et al., 2024). Researchers have suggested several reasons for this limited acceptance, including limitations surrounding the instructional time required to effectively support students with difficulties (Warshauer, 2015), or students' difficulties in effectively solving problems (Daroczy et al., 2015; Scheibling-Sève et al., 2020; Wahyun et al., 2023), knowledge that recent research has used to seek different ways of approaching problem solving in relation to the student and the teacher (Achmetli et al., 2019; Ke & Clark, 2020; Saadati et al., 2019; Saadati & Reyes, 2019).

Students' experiences and behaviours in problem solving reflect and become a way of thinking consistent with their mathematical practices, which are manifested in terms of the activities they carry out throughout all phases of problem solving (Santos-Trigo, 2024). Empirical evidence shows the difficulties students encounter when solving problems associated with understanding quadratic functions as a mathematical object of study (Fitrianna & Rosjanuardi, 2021; Graf et al., 2018; Mutambara et al., 2020; Reid O'Connor & Norton, 2024; Sebsibe & Abdella, 2025; Selvan, 2025), which are manifested in the errors students make when solving mathematical tasks. Students who are confused by the algebra associated with quadratic functions in secondary school are likely to experience challenges when confronted with the more abstract algebra that underpins calculus (Pyzdrowski et al., 2013). The study and analysis of difficulties is a primary source of feedback in the teaching-learning process. From the information provided by these processes, teachers and researchers must extract guidelines that can be used to improve classroom processes and educational approaches that can be designed. However, studies focus mainly on the manifestation of difficulties in mathematics, that is, on students' mathematical errors (Baybayon & Lapinid, 2024; Díaz & Flores del Río, 2022; Matitaputty et al., 2025; Tendere & Mutambara, 2020; Thomas & Mahmud, 2021). Studies on the trinomial of problem solving, quadratic functions and difficulties are less common.

According to the OECD (2023), problem solving helps individuals understand the role that mathematics plays in the world and make the well-informed judgements and decisions required of thoughtful, constructive and committed citizens of the 21st century. However, in the latest standardised PISA 2022 assessment, overall mathematics results for OECD countries showed a significant drop of 15 points compared to 2018, reversing the progress made in the previous decade and causing many students to score below the basic level of competence. Although the PISA 2022 results place Chile as the leader in Latin America and the Caribbean in the three areas assessed, performance in mathematics was below the average for OECD countries. From 2006 to 2022, nearly half of students in Chile did not achieve the score required to reach performance level 2 (basic level), which would demonstrate that they have the minimum skills required to enter the labour market or pursue higher education. In the results for mathematical reasoning processes related to specific cognitive processes that are executed when facing a mathematical task, Chilean students show relative strength in the process of interpreting and evaluating. On the contrary, the process of formulating, that is, translating situations from the real world into an abstract mathematical concept and identifying a calculation or strategy to solve it, is more difficult for students in this country.

In mathematics, errors and difficulties are intrinsically linked The mistakes made by students are a manifestation of the difficulties they face in learning (Kurudirek et al., 2025; Yap & Wong, 2024). Detecting and analysing these difficulties is crucial for identifying problem areas and adapting teaching strategies.

In this context, a study was conducted with the aim of assessing performance by analysing levels of progress in solving different types of problems according to their nature and context, and identifying the types of difficulties that arise when solving problems involving the application of quadratic functions among secondary school students in two regions of southern Chile. To this aimed, the following research questions were formulated: (1) What types of problems are most difficult to solve? and (2) What types of difficulties are most commonly encountered in solving problems contextualised to the quadratic function?

2. METHOD

The development of this study corresponds to descriptive-quantitative research, focused on the percentage distribution of responses for each variable, in addition to using non-parametric inferential statistical techniques due to the nature of the data, which comes from ordinal and nominal categorical variables, using the non-parametric Mann-Whitney U and Kruskal-Wallis tests and contingency tables (Hernández-Sampieri & Mendoza, 2018).

2.1. Participants

The population consists of students from public educational establishments in two regions of southern Chile (Region 1 and Region 2). This study analyses 304 third-year secondary school students (n=148 Region 1; n=156 Region 2) who make up the intentional non-probabilistic sample, taking into account the feasibility of applying the assessment instruments in intact courses which, according to the curriculum, were familiar with the quadratic function. In general, participants had an average age of 16, of whom 59.2% were female. Before beginning the research, the objective of the study and the form of participation as a student were explained, indicating, in addition, that the test was anonymous and that express consent was required to participate.

2.2. Instrument

The instrument used was a test to solve mathematical problems contextualised to the quadratic function. Students had two teaching hours to complete the test, which was administered in their respective classrooms during normal mathematics class time, following informed consent.

A test previously validated by Díaz and Flores (2025) was chosen, consisting of open-ended problems, classified according to Díaz and Poblete (2001) and presented in Figure 1.

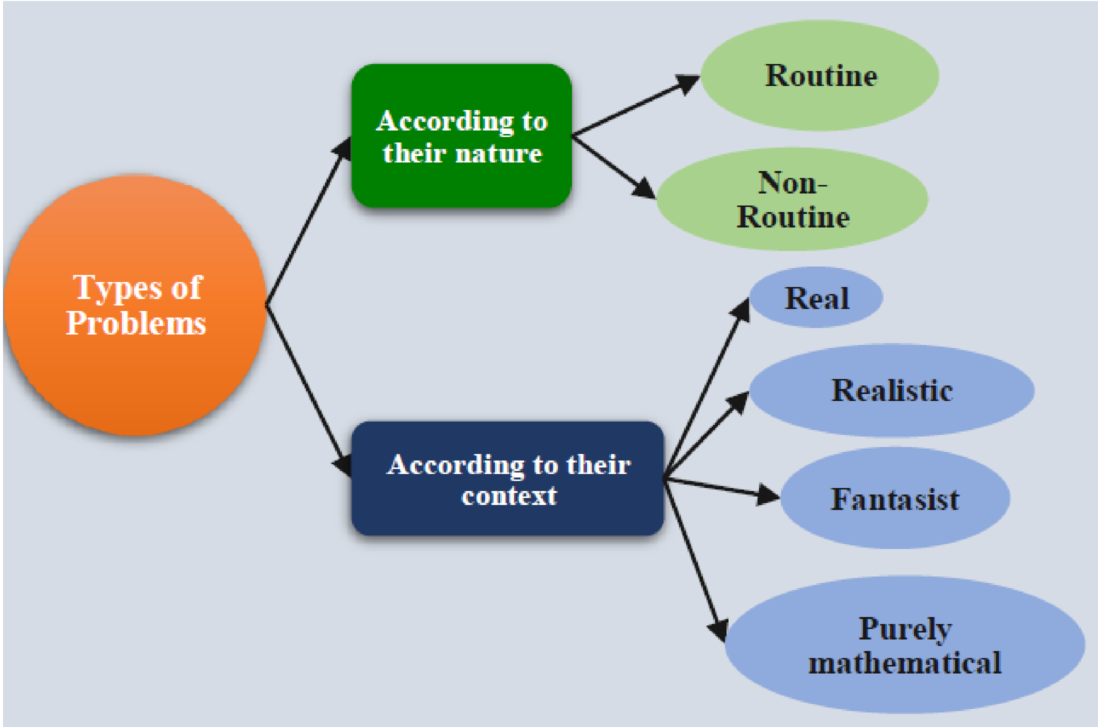


Figure 1. Types of mathematical problems

Routine Problem Solving

This includes posing, formulating, and solving real-world, realistic, fantasist, and purely mathematical problems that require establishing connections in order to solve them. Routine problems are similar to those solved during instruction courses; the student follows a sequence that involves understanding concepts and algorithms to arrive at valid solutions.

Real-life context problem: A context is real if it actually occurs in reality and involves the student's actions in that context. Example: "*Construct a function whose numerator is a constant value and whose denominator is a quadratic function. Find its asymptotes in detail and indicate the position of the curve with respect to them*".

Realistic context problem: A context is realistic if it is likely to actually occur. It is a simulation of reality or part of reality. Example: "*Gravity on the Moon is roughly one-sixth that of Earth. Suppose an astronaut is on the Moon, standing on a hill 60 meter high. If he jumps upwards at a speed of 40 meter per second, how long will it take him to touch the ground at the foot of the hill?*"

Problem of fantasist context: A context is fantasist if it is the product of imagination and has no basis in reality. Example: "*112 dinosaurs were introduced to an island. At first, they reproduced rapidly, but the island's resources began to dwindle and the population declined. The number of dinosaurs t years after they were left on the island is given by: D(t)= –* $t^2+22t+112$*, (t >0). Determine: a) The number of years during which the dinosaur population increased. b) At what point did the dinosaur population become extinct?*"

Purely mathematical context problem: A context is purely mathematical if it refers exclusively to mathematical objects: numbers, relationships and arithmetic operations, geometric figures, etc. Example: "*Calculate the diagonal of a rectangle knowing that the base is equal to three-quarters of the height and that the area is 48*"

Non-Routine Problem Solving

A problem is considered non-routine when a student does not know a previously established answer or procedure, or routine, to find it. Example: "*The distributor of Phantom 3 drones has determined that it sells an average of 300 drones per month if the unit price is 100 UM. It has estimated that for every 5 UM decrease in price, sales will increase by 25 drones per month. What price should be set for each drone to obtain the maximum monthly income?*"

The distribution of the problems in the test was as follows: Problem 1 (P1) routine with a realistic context, P2 routine with a realistic context, P3 routine with a fantasist context, P4 non-routine, P5 routine with a realistic context, P6 routine with a fantasist context, P7 routine with a realistic context, P8 routine with a purely mathematical context, P9 routine in a purely mathematical context, P10 routine in a fantasist context, and P11 routine in a realistic context. The students' level of achievement was determined using the Rash Model (adapted by Díaz & Poblete, 2019). This model is associated with a five-point scale, which indicates the level of progress towards the correct solution to the problem. The scoring scale records every detail of the solver's attempt to find the solution (see Table 1).

Table 1. Problem-solving grading scale

Score	Solution stages
0	No Start: The student is unable to start the problem or delivers work which is meaningless
1	Focus: The student focuses the problem with a meaningful work, indicating comprehension of the problem, yet faces difficulties easily.
2	Substance: Sufficient details show that the student has been oriented to a rational solution, yet relevant error s or wrong interpretations prevent the process of the correct resolution.
3	Result: The problem is about to be resolved, yet few mistakes lead to a wrong final solution.
4	Completion: The proper method has been used and it has led to the correct solution.

To analyze the difficulties in problem solving, we considered the classification proposed by Di Blasi Regner et al. (2003), which includes difficulties associated with cognitive domain: complexity of mathematical objects, complexity of mathematical thinking processes, and complexity of teaching processes, which are presented:

(1) Difficulties associated with the complexity of mathematical objects: mathematical objects are communicated, mainly in written form, through mathematical symbols with the help of everyday language, which facilitates the interpretation of these symbols. We thus encounter various conflicts associated with the understanding and communication of mathematical objects. One of these conflicts arises from the assistance that everyday language provides in interpreting mathematical symbols. The language of mathematics is

precise, subject to exact rules, and does not communicate its meaning except through the exact interpretation of its symbols. This conflict involved in the use of ordinary language within the mathematical context is a conflict of precision.

(2) Difficulties associated with mathematical thought processes: these are evident in the logical nature of mathematics and in the breaks that necessarily occur in relation to modes of mathematical thought. The formal deductive aspect has always been considered one of the main difficulties in learning mathematics, with formal proofs being abandoned in some secondary school mathematics programs. However, this does not mean that logical thinking has been abandoned, i.e. the ability to follow a logical argument, and this inability is one of the causes that creates the greatest difficulty in learning this science.

(3) Difficulties associated with teaching processes: these relate to the school institution, the mathematics curriculum and teaching methods. The school institution must promote a school organization that tends to reduce the difficulties of learning mathematics depending on the curriculum materials, resources and teaching styles. This organization affects both the spatial-temporal elements and the grouping into homogeneous or heterogeneous classes, according to their abilities in mathematics.

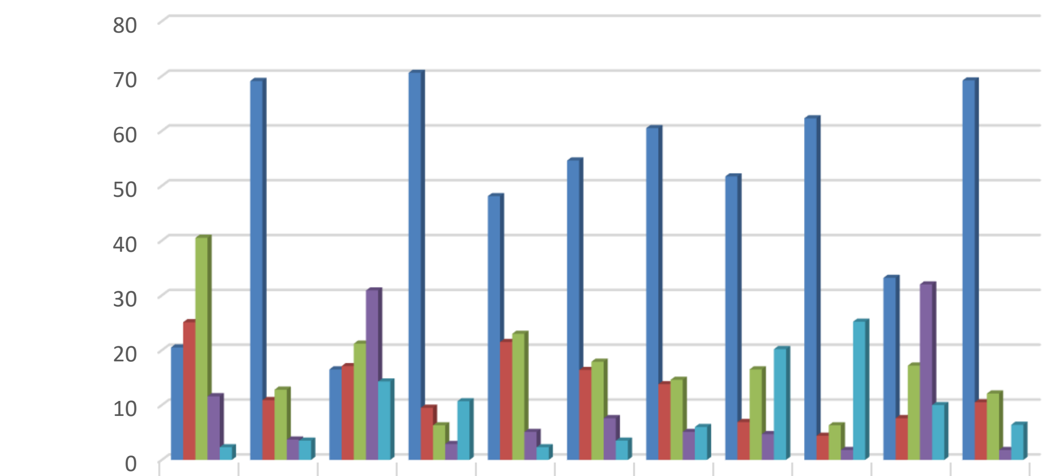
The organization of the mathematics curriculum can give rise to various difficulties in learning the subject. There are four basic elements to consider as difficulties in the mathematics curriculum: the skills needed to develop mathematical abilities that define a student's competence in this science, the need for prior knowledge, the level of abstraction required, and the logical nature of school mathematics.

The reliability of the mathematics test was measured using the Kuder-Richardson coefficient (KR-20) and was equal to 0.92. The reliability for difficulties associated with the complexity of mathematical objects (difficulty 1) was 0.78; difficulties associated with mathematical thinking processes (difficulty 2) was 0.83; and difficulties associated with teaching processes (difficulty 3) was 0.85. Finally, the homogeneity of the sample was confirmed using the non-parametric Mann Whitney U test with a significance level of 5%. Data analysis was performed using the SPSS statistical package (version 30).

3. RESULTS AND DISCUSSION

3.1. Results

To address the research's objectives, we analyzed the students' performance and the cognitive difficulties evidenced in solving the eleven problems contextualized to the quadratic function and associated with the complexity of mathematical objects, mathematical thinking, and the teaching process. Figure 2 shows the total results in percentage terms of the performance in the test of solving types of problems involving quadratic function applications by secondary school students belonging to Region 1 and Region 2.



	P1	P2	P3	P4	P5	P6	P7	P8	P9	P10	P11
No Start	20,5	69,1	16,5	70,6	48,1	54,6	60,5	51,7	62,3	33,2	69,2
Focus	25,1	10,9	17,1	9,5	21,5	16,4	13,8	6,9	4,4	7,6	10,5
Substance	40,5	12,8	21,2	6,3	23	17,9	14,6	16,5	6,3	17,2	12,1
Result	11,6	3,7	30,9	2,9	5,1	7,6	5,1	4,7	1,8	32	1,8
Completion	2,3	3,5	14,3	10,7	2,3	3,5	6	20,2	25,2	10	6,4

Figure 2. Percentage distribution of problem-solving performance

When analyzing the levels of progress in solving problem types according to nature and context, it can be seen that the highest percentage corresponds to the No Start level and is associated with non-routine, purely mathematical and realistic problem types. This indicates that students are unable to begin the problem or submit work that has no meaning whatsoever. It is evident that problem (P4), which was non-routine in nature, was the one that caused the most difficulty for students, mainly reaching the first level of difficulty with 70.6%. Both the Focus and Substance achievement levels agree on the type of problem with the highest percentage representation, which is Realistic (P1) with 25.1% in the Focus achievement level and 40.5% in Substance; fantasist (P3) with 17.1% in Focus and 21.2 in Substance. In particular, both achievement levels indicate that students demonstrate knowledge of problem solving, but significant errors or misinterpretations prevent them from arriving at the correct solution. For its part, the highest level of achievement, called Completion, represents the percentage of students who managed to use an appropriate method and produced a correct solution, achieving 25.2% in routine problems of a purely mathematical nature.

On the other hand, the highest performance levels, considering the two levels of progress (Results and Completion), are evident in problems 3 (P3) and 10 (P10), both of a routine nature and with a fantasist context, with 45.2% and 42% achievement respectively.

These correspond to problems that are a product of the imagination and have no basis in reality. Next are problems P9 and P8, with 27% and 24.9% achievement respectively, both routine and purely mathematical in context, i.e., problems that refer exclusively to mathematical objects: numbers, relationships and arithmetic operations, geometric figures, etc. Figure 3 shows the percentage distribution of the types of difficulties observed by region.

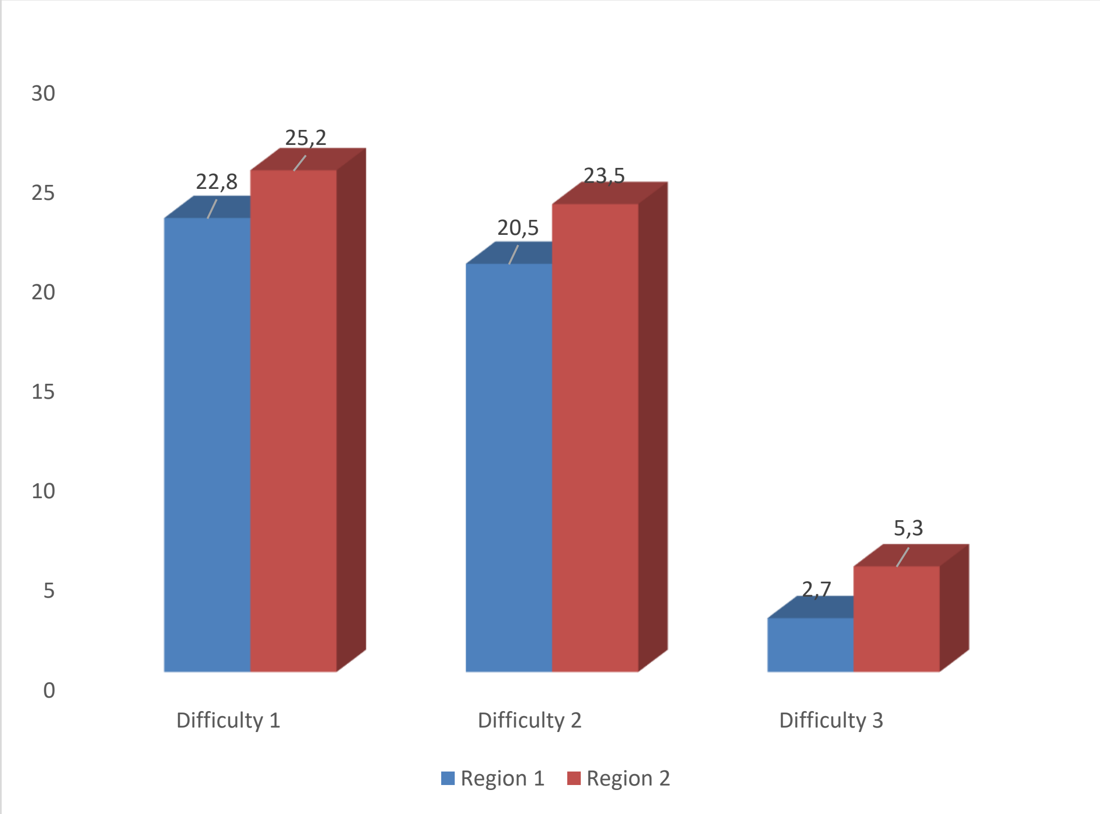


Figure 3. Percentage distribution of types of difficulties in problem solving

When analyzing the types of difficulties manifested in errors made by students in solving open-ended problems, the majority, averaging 24% between the two regions studied, correspond to difficulties associated with the complexity of basic mathematical objects (difficulty 1) that arise in the operational and conceptual status of the quadratic function. Solving the problems required prior general knowledge of the subject and, in particular, of analytical expressions. The results indicate that students did not find it easy to equate expressions and operate with them to obtain the solution; they did not understand the meaning of expressing a parameter as a function of other parameters.

They also demonstrated technical difficulties in symbolic manipulation and interpretation of results, which may be due to the fact that they routinely work with concrete examples and only literal expressions, as evidenced by their verification using numerical examples. An average percentage of 22% obtained between the two regions revealed difficulties associated with the breaks that necessarily occur in relation to modes of mathematical thinking (difficulty 2).

A much smaller percentage of difficulties are associated with teaching processes related to the school institution, the curriculum and the methods used to teach mathematics (difficulty 3), which averaged 4% in both regions. This can be explained by the fact that educational institutions promote a school organization that tends to reduce the difficulties of learning mathematics through the use of appropriate curriculum materials, resources and teaching styles. On the other hand, teaching methods must be linked both to the organizational elements of the school institution and to the curriculum organization. There are several aspects to consider, such as language, which must be adapted to the abilities and comprehension of the students; the sequencing of learning units, which must be adapted to the internal logic of mathematics; respect for individuality, which has to do with the pace of work in class; resources; and adequate representation.

To determine whether there are any significant differences between the types of difficulties detected in problem solving according to the region of the educational establishment, the non-parametric Kruskal-Wallis test is used (see Table 2).

Table 2. Significant differences in difficulties versus region

Contrast Statistics [a,b]			
	Difficulty 1	**Difficulty 2**	**Difficulty 3**
Chi-square	13.092	27.649	53.530
gl.	2	2	2
Sig.asintót.	0.001	0.000	0.000

a. Kruskal-Wallis test

b. Grouping variable: Region of educational establishment

Based on a post-hoc Mann-Whitney U test, Table 3 shows the differences according to the type of difficulty between regions, showing the significance obtained for each mathematical problem.

Table 3. Differences between difficulty and problem

Types of problems	No problem	Difficulty 1	Difficulty 2	Difficulty 3
Realistic	P1	0.167	0.211	0.126
	P2	0.053	0.616	**0.017**
	P5	0.103	0.088	0.103
	P7	0.689	**0.026**	0.340
	P11	0.738	0.077	0.151
Fantasist	P6	0.523	0.100	0.331
	P3	0.445	0.316	0.381
	P10	0.534	0.378	0.584
Non-rutine	P4	0.074	0.616	**0.007**
Purely mathematical	P8	0.147	0.473	0.328
	P9	0.567	0.293	0.328

Significant differences are observed in two difficulties: the first is in mathematical thinking processes (difficulty 2), specifically in problem (P7) (sig=0.026), where the type of problem addressed is realistic in context; The second and last corresponds to difficulties associated with teaching processes (difficulty 3), specifically in problem (P2) (sig=0.017) and (P4) (sig=0.007), which correspond to realistic and non-routine problem types, respectively.

3.2. Discussion

This research linked the ability to solve contextualized problems involving quadratic functions to the difficulties faced by secondary school students when asked to demonstrate this skill, which is considered by the literature to be one of the high-level and fundamental abilities required for the acquisition of mathematical knowledge. Based on this research, students exhibit a very low level of achievement in problem-solving, as noted by other researchers and institutions (Csapó & Funke, 2017; Lozada & Díaz-Fuentes, 2018; OECD, 2023) and experience various difficulties in solving problems associated with quadratic functions (Azzolina et al., 2019; Esquer et al., 2017; Ruli et al., 2018). With regard to the types of problems according to their nature and context, students demonstrate greater ability when solving routine problems and problems with a purely mathematical context, which refer exclusively to mathematical objects such as numbers, relationships and arithmetic operations, geometric figures, etc. However, an important result was obtained in solving routine problems with fantasist contexts, which are the product of the imagination and have no basis in reality, but were formulated in mathematical contexts that the students approached and solved more effectively, achieving levels of achievement twice as high as those for purely mathematical context problems, as has been verified in other mathematical objects by Díaz and Flores (2025), and Díaz et al. (2025).

The results obtained lead us to conclude that recognizing the usefulness of problem solving is not enough. In the context of formal education, students will also encounter problems in mathematics. These problems may arise from mathematics itself, or they may arise from everyday life, involving facts and contexts that can be modelled in mathematics, as Palmer (2018) suggests, or they may be the product of imagination, as in the case of fantasist context problems, as Díaz (2020) suggests.The data from our study showed that when faced with non-routine problems, students tend to incorrectly apply memorized procedures rather than modify them or develop new solutions. One possible reason for this difficulty may be associated with the instructional approach in secondary education, which emphasizes memorization and the application of routine procedures. In this way, students become more competent in problems that execute routine algorithms than in solving new problems that they have not practiced solving, which is consistent with our findings and those of Gavaz et al. (2021) and Nguyen et al. (2020).

A math teacher who only includes routine problem solving diminishes students' interest and even limits their intellectual development. However, if this teacher increases students' interest through problem solving in different contexts and of a stimulating nature, then the teacher will instil in students a sense of belonging to mathematics, understanding and independent thinking. These conclusions are consistent with those found in Polya (1981) and Simamora et al. (2019).

Test analysis provides a better understanding of difficulties based on mistakes made by students when solving contextualized problems involving quadratic functions. It has been found that the difficulties encountered by most secondary school students are primarily associated with the complexity of mathematical objects and, secondarily, with difficulties associated with mathematical thinking processes. In a very small percentage of cases in both regions, difficulties associated with teaching processes were observed. With regard to the complexity of mathematical objects, written communication mainly uses symbols together with everyday language to make it more understandable. This system, called mathematical language, includes not only technical symbols, but also grammatical conventions that are specific to mathematics and elements of everyday language, achieving with both a communication of mathematical ideas that avoids ambiguities, allows complex ideas to be expressed briefly and directly, and facilitates the expression of complex thoughts in a simpler way.

The difficulties associated with mathematical thinking processes were observed in non-routine problems that require complex and higher-order thinking skills, coinciding with Keleş and Yazgan (2025), Montague et al. (2014), and Niss (2015) in stating that the context of problems can vary from experiences that are familiar to students to abstract concepts involved in more complex problems such as non-routine ones.

Difficulties were also observed in mental arithmetic, logical reasoning, and comprehension. Although students demonstrate knowledge of how to apply the quadratic function formula, they fail to understand that there is a single maximum or minimum point for this mathematical object, which is associated with phenomena of change. They also had difficulty differentiating between dependent and independent variables. In general, they did not achieve effective resolution of the types of problems, which was limited to the algorithmic application of a more basic algebraic procedure (application of additive and multiplicative inverses, solving the equation, etc.). When faced with the possibility of working on the interpretation of the quadratic function as a mathematical object, students tend to develop algebraic and memorization processes, which limits the development of problem-solving skills. In order to overcome these difficulties, greater intervention on the part of the teacher is required, but also joint work with the student, since this type of error reveals a lack of both theoretical and conceptual content, in line with the research of Agustyaningrum et al. (2018).

4. CONCLUSION

The results of this study revealed that the greatest cognitive difficulties in solving problems contextualised to the quadratic function were found in the complexity of mathematical objects and in mathematical thinking processes. In the types of problems that address the nature and context in which they are posed, the highest performance was found in routine problems and those with a fantastical and purely mathematical context, while the lowest achievement was found in non-routine problems.

Our findings support the fact that a difficulty is an idea that, at the time of concept formation, has been effective in dealing with previous problems, but which proves to be a failure when attempted in a new situation. Despite the failure, attempts are made to salvage it, ultimately becoming a barrier to further learning. This research provides us with an assessment tool that will allow us to continue analyzing why students make difficulties and how this affects their work in mathematics in general and problem solving in particular.

Declarations

Author Contribution : VD: Conceptualization, Data curation, Formal analysis, Investigation, Methodology, Validation, Visualization, Writing - original draft, and Writing - review & editing.

Funding Statement : This research did not receive any funding, either public or private.

Conflict of Interest : The authors declare no conflict of interest.

Additional Information : Additional information is available for this paper.

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