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EVALUATING GRAPHING QUADRATIC WORKSHEET ON VISUAL THINKING CLASSIFICATION: A CONFIRMATORY ANALYSIS

Rina Oktaviyanthi*, Ria Noviana Agus
Universitas Serang Raya, Indonesia

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ABSTRACT

Applying a graphing quadratic worksheet as a medium for learning the concept of a Quadratic Function clearer is an alternative instrument to accommodate the needs of developing students' mathematical visual thinking. In implementing graphing quadratic worksheet should show details of the dominant and recessive visual thinking classification aspects that develop in students. Classification of dominant and recessive aspects of visual thinking needs to be completed to determine stages in improving the worksheet and learning instructions that are applied especially to recessive aspects. Therefore, there is a need to evaluate the factors and trace the classification aspects of visual thinking that developed in students after practicing the graphing quadratic worksheet. The purpose of this research is to determine the categorization aspects of visual thinking in graphing quadratic worksheet items that develop and do not develop in students. Confirmatory factor analysis was employed as a research method on 12 sub-variables from the three classifications of visual thinking. As research data, 93 student records were used. Four main factors were formed as a result of the confirmatory factor analysis procedure, with the top two factors, namely factors 1 and 2, completely representing each aspect of the visual thinking classification and achieving the factor loading significance criteria. The implication is that the variables developed in the graphing quadratic worksheet for each aspect of the visual thinking classification have a strong relationship with the visual thinking ability overall. Enhanced by a cumulative variance value for factors 1 and 2 specifically 56.88% of the total 81.78% for all factors. Thereby it can be said that the categorization aspect of visual thinking that develops in students after implementing a graphing quadratic worksheet is achieved sensibly.

Corresponding Author:

Rina Oktaviyanthi,
Department of Mathematics Education,
Universitas Serang Raya
Jl. Raya Cilegon Km. 5, Taktakan, Serang City, Banten 42162, Indonesia.
Email: rinaokta@unsera.ac.id

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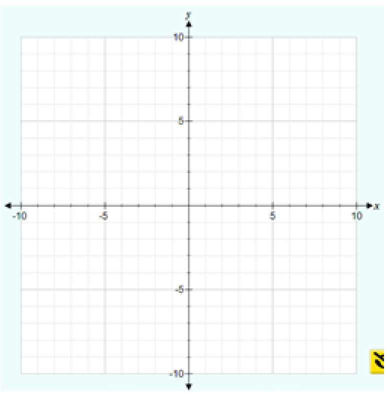
1. INTRODUCTION

The Quadratic Function is one of the abstract mathematical concepts that students find challenging to learn. Numerous research studies have revealed that mathematical visual thinking can address students' difficulties in learning Quadratic Functions (Agus & Oktaviyanthi, 2023; Presmeg, 2020). Students with mathematical visual thinking abilities are more potentially to be proficient in procedures for transforming information into other mathematical situations (Frick, 2019). This mathematical situation promotes positive progress regardless of whether students are solving problems, but also when mathematical concepts are being constructed in their minds (Hawes & Ansari, 2020). According to Heng and Said (2020) and Bjorklund (2022), learning through visual media can assist learners' process and understand a concept skilfully as a consequence of visual stimulation influences their cognition area. Conforming to Elsayed and Al-Najrani (2021), mathematical visual thinking represented by a diagram or scheme is not merely a picture or illustration, but an accurate depiction of the quantity and relationship of certain mathematical problems. The goal is for motivating students to learn more, involving in the discovery of implicit mathematical ideas, supporting in reducing cognitive load, and improving higher-order thinking processes (Anmarkrud et al., 2019; von Thienen et al., 2021). The description's points confirm the importance of developing mathematical visual thinking in students.

Conceptually, visual thinking is expressed by Cain (2019) and Fernández-Fontecha et al. (2019) as perception and discrimination, interpretation of the existence a thing (shape and object), and organizing mental images in various different modes through several processes such as deletion, addition, reflection, rotation and cutting, then working to find a relationship and translating it into positions and literal symbols to reach a conclusion. Operationally, Elsayed and Al-Najrani (2021) state visual thinking is the ability to transform information of all kinds into images, graphics, or other forms that can help communicate information. The classification of visual thinking is divided into three abilities: (1) the skill of visual discrimination (VD), the ability to detect differences and classify objects, symbols, or shapes (positions and patterns); (2) the skill of visual perception (VP), the ability to organize and interpret the information seen and to give meaning; and (3) the skill of visual analysis of shape (VS), the ability to connect abstract representations into concrete understanding or vice versa (Elsayed & Al-Najrani, 2021; Hermann & Klein, 2015; Lin, 2019). The following illustrates the graphing quadratic worksheet used in the study (see Figure 1).

(a) If $a > 0, b > 0, c > 0$



Create a quadratic function equation based on the conditions!

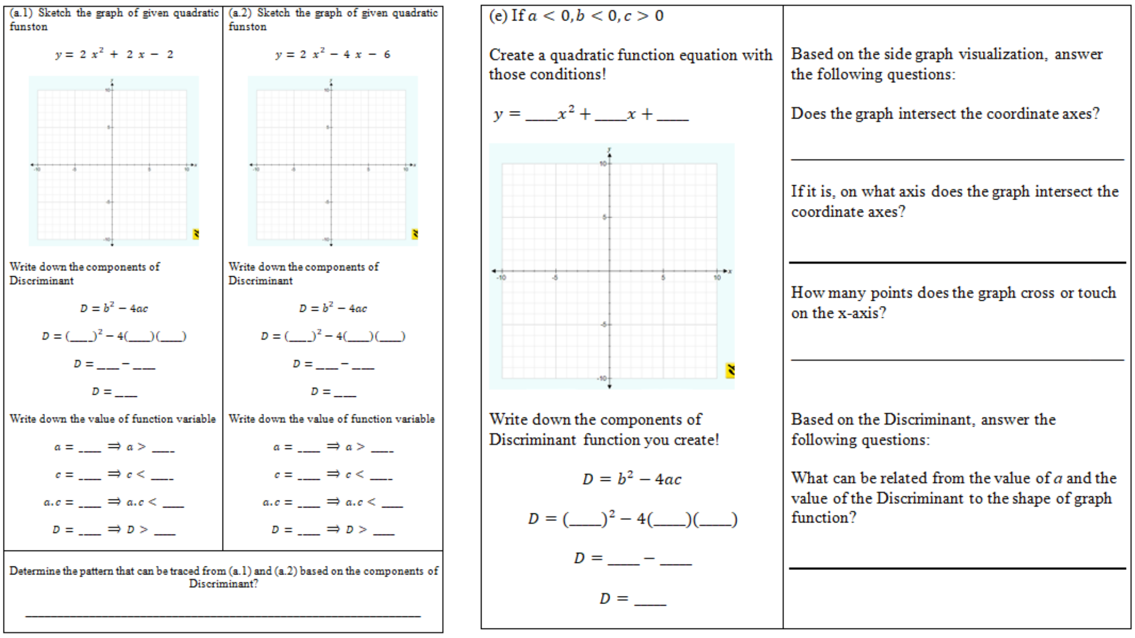
$y =$ ____$x^2 +$ ____$x +$ ____

Sketch the curve on the Phet Simulation coordinate plane!

What can be connected from conditions a, b and c to the shape of the resulting curve?

____

(a)



(b) (c)

Figure 1. Illustrative worksheet: (a) VD aspect, (b) VP aspect, (c) VS aspect

Adopting a graphing quadratic worksheet as an illustrative medium for learning the concept of Quadratic Functions viewed appropriate based on the instrument's feasibility test to facilitate the future requirements of students' mathematical visual thinking (Agus & Oktaviyanthi, 2023). The results of incorporating a graphing quadratic worksheet on 93 first-year students registered for Calculus at Universitas Serang Raya demonstrate that the instrument's validity and reliability values exceed the standard value (Agus & Oktaviyanthi, 2023). These conditions affirm that the questions or statements compiled in the graphing quadratic worksheet satisfy the visual thinking classification achievement indicator's appropriateness value (Oktaviyanthi & Agus, 2021; Shanta & Wells, 2022). Elsayed and Al-Najrani (2021) elaborated the classification of visual thinking includes the skills of visual discrimination, visual perception, and visual analysis of shapes. However, the results of the graphing quadratic worksheet implementation do not provide information on which visual thinking classification aspects are dominant and recessive in students. Furthermore, the implementation results do not indicate how consistent the classification aspects of visual thinking appear to students. In fact, for the purpose of optimizing students' mathematical visual thinking, teachers should first understand which aspects of visual thinking students have underdeveloped aiming to determine steps to improve the instruments and learning instructions employed.

Ghazali and Nordin (2019) and Vucaj (2022) both recommend conducting a comprehensive examination of an instrument or model with a focus on obtaining accurate information about the pattern of variables that appear or the interdependence of observed variables. Factor analysis is commonly used in such techniques. Brown (2015) defines factor analysis as a statistical technique for determining the interdependence of a structure (factor or dimension) of several variables observed simultaneously in time. The primary purpose of factor analysis is to simplify or reduce variables into smaller number of dimensions (Crede & Harms, 2019; Hox, 2021)There are two types of factor analysis: confirmatory factor analysis (CFA), which is used to test the effects of methods or to construct validation from evaluation measurements, and exploratory factor analysis (EFA), which is used to investigate general indicators into specific indicators (Hox, 2021; Jiang & Kalyuga, 2020).

Confirmatory factor analysis (CFA) was utilized to determine how established the visual thinking aspect of students was upon completion of a graphing quadratic worksheet. This is coherent with Mustafa et al. (2020) who asserts that confirmatory analysis can be used to investigate the role of a variable on the indicators that constitute it.

More studies have concentrated on the application of confirmatory factor analysis, particularly on the evaluation of an instrument's or variable's factors. On the strength of confirmatory factor analysis is appropriate for verifying the factor structure of a series of observed variables and allowing researchers to test the hypothesis of whether there is a relationship between the observed variables and the latent variables that emerge (Brown, 2015; Hox, 2021). Ayebo et al. (2019) tested the psychometric characteristics and arrangement of the student attitude questionnaire towards statistics courses in several studies related to confirmatory factor analysis in 2019-2022. The research of Zainudin et al. (2019) then measures the construct validity of students' mathematical creativity assessment instruments with a view to minimize assessment gaps. Furthermore, Asempapa and Brooks (2022) research employs confirmatory analysis in the development of quantitative instruments to assess mathematics teachers' attitudes toward the practice of mathematical modeling. Meanwhile, Kaplon-Schilis and Lyublinskaya (2020) scrutinize predictors of technological pedagogical content knowledge in teacher preparation programs as an assessment of the effectiveness of technology-assisted learning. The design was then evaluated and validated by González-Ramírez and García-Hernández (2021) for the level of student satisfaction with the mathematics learning system at universities. Alwast and Vorhölter (2022) also investigate the development of video-based instruments to ascertain teacher competence in the context of mathematical modeling. Sari et al. (2022) then developed and determined construct validity for mathematical reasoning and proof instruments. Several recent studies on confirmatory factor analysis confirm that this statistical technique is beneficial in assessing factors or testing instruments.

Derived from previous research, the use of confirmatory factor analysis was not found to confirm whether or not the visual thinking classification aspect was uniform in the graphing quadratic worksheet that students developed. The purpose of this research is to investigate aspects of visual thinking classification that develop in students after using a graphing quadratic worksheet, based on the exploration of research problems and in accordance with reviews of several studies related to confirmatory factor analysis in 2019-2022. It is expected that the research findings will not only fill a gap in the assessment of students' visual thinking skills, but will also contribute to improving the evaluation system for teaching and learning activities, both in terms of learning instruments and teaching instructions.

2. METHOD

To achieve the research objective of identifying aspects of mathematical visual abilities that develop with the implementation of a graphing quadratic worksheet, a quantitative research approach with a confirmatory factor analysis measurement model has been used. Using a graphing quadratic worksheet, analysis activities were conducted to obtain student performance data in studying the concept of Quadratic Functions (Agus & Oktaviyanthi, 2023). Student performance is obtained by completing worksheet with a rating for each visual thinking classification item. All items in the aspect of visual discrimination is 5, while every item in the aspect of visual perception and visual analysis of shapes is 10, then the total value in the worksheet is 100. This factor analysis study included 93 first-year students who had enrolled in Calculus I courses at the University of Serang Raya.

The research variable is a graphing quadratic worksheet indicator that corresponds to the visual thinking classification (see Table 1).

Table 1. Graphing quadratic worksheet indicators

Classification of Visual Thinking	Sub Variable	Description of the Graphing Quadratic Worksheet Scheme
The skill of visual discrimination. Student's ability to identify differences or similarities in the form of a representation of a particular mathematical concept (VD)	VD1	If a > 0, b > 0 and c > 0, create a quadratic function equation based on these conditions and then sketch the curve on the Phet Simulation coordinate plane. What can be connected from conditions *a*, *b* and *c* to the shape of the resulting curve?
	VD2	Curve position of a quadratic function $y = ax^2 + bx + c$ is known. Based on this information, determine the possible position of the curve if the value of *a* is positive (+), the value of *b* is negative (-) and the value of *c* is positive (+)?
	VD3	Given a quadratic function $= 6x^2 + 5x + 4$ and $y = -6x^2 + 5x + 4$. Sketch the curve on the Phet Simulation coordinate plane. Write down what is observed from the values *a*, *b* and *c* in each quadratic function with the shape of the curve.
	VD4	The curve position of the quadratic function with values *a* > 0 and *a* < 0 is shown. What facts can be found from the information on the shape of the curve and the value of *a*?
The skill of visual perception. Students' ability to investigate the implicit form of a certain mathematical concept (VP)	VP1	Given a quadratic function $y = 2x^2 + 2x - 2$ and $y = 2x^2 - 4x - 6$. Sketch the graph and write down the components that make up the Discriminant. Determine the pattern that can be traced from the two quadratic functions based on the components of the Discriminant.
	VP2	Known the quadratic function and its graphical form. Observe and write down the value of the Discriminant constituent, the value of the Discriminant and the value of a quadratic function. What is the confirmed regularity of the information?
	VP3	Three equations of quadratic functions and their graphs are known. Determine the Discriminant value and square root of the three functions. What can be validated from the discriminant value relationship and the square root of the function obtained?
	VP4	Various positions of the quadratic function graph are shown for values *a* > 0 and *a* < 0. Determine the discriminant value of each graph.
The skill of visual analysis of shapes. Student's ability to determine part of the overall mathematical concept shown or vice versa (VS)	VS1	Create a quadratic function equation with the conditions *a* < 0, *b* < 0 and *c* > 0. Sketch the graph and calculate the value of the function's discriminant. Does the graph intersect the coordinate axes? How many points does the graph cross or touch on the *x*-axis? What can be related from the value of *a* and the value of the Discriminant to the shape of the graph of the function?
	VS2	Know the quadratic function and its graphical form. Determine the value of the Discriminant constructor, the discriminant calculation value, the intersection axis of the graph and the number of points through which the graph passes. What patterns can be identified from the information?
	VS3	Given are six different quadratic functions along with their graphical representations. Write down the criteria for *a* and *ab* values in the provided column. What can be identified from the value of *a* and *ab* to the position of the function graph?
	VS4	The various positions of the graph of the quadratic function for values *a* > 0 and *a* < 0 are shown. Write down for each graph whether it intersects or touches the coordinate axes.

The procedure for the confirmatory factor analysis measurement model in this study refers to the explanation of Hair et al. (2014) and the calculation process using SPSS assistance goes through the stages below:

a. Testing the assumptions of factor analysis using *Bartlett's Test* and KMO which are the main component tests. The *Bartlett test* hypothesis namely:
   H_0: the correlation matrix is an identity matrix.
   H_1: the correlation matrix is not an identity matrix.

   Rejecting criterion H_0 is if the probability value (Sig.) *Bartlett's Test* < 0.05 with the understanding that the correlation matrix is not an identity matrix so that principal component analysis can be confirmed. While in the KMO test, it is said to be middling if it has a KMO value range of $0.7 \leq \text{KMO} < 0.8$ (see Table 2).

Table 2. Classification of KMO values (Arpaci, 2019)

KMO value	KMO level	KMO value	KMO level
> 0.9	Marvelous	0.6 – 0.7	Mediocre
0.8 – 0.9	Meritorious	0.5 – 0.6	Miserable
0.7 – 0.8	Middling	< 0.5	Unacceptable

b. Calculating the MSA (*Measures of Sampling Adequacy*) value for each variable with the condition that if the MSA value is < 0.5 then the variable cannot be analyzed further (see Table 3). The meaning of this value is the weak correlation between variables which has implications for reducing the variables and then re-analyzing the remaining variables.

Table 3. MSA acceptance criteria (Deutsch & Beinker, 2019)

MSA value	MSA level
= 1.0	A variable can be predicted without error by other variables.
> 0.5	A variable can still be predicted and analyzed further.
< 0.5	A variable is unpredictable and cannot be analyzed further, so it must be reduced or removed from the model.

c. Performing factor extraction using *principal component analysis* which can be seen from the value of *communalities* in the SPSS calculation output. *Communalities* values < 0.5 are considered factors that are unable to explain indicators or variables.

d. Determining the number of main factors or components that are formed through *eigenvalue*, cumulative variance or *scree plot* at the SPSS calculation output. The main components that are worth choosing are those with an *eigenvalue* > 1 (Chatfield, 2018) or those with a cumulative variance of more than 80% (Matteson & James, 2014).

e. Examining the factors that are formed (*loading factor*) by rotating the factors using *varimax rotation*. The significance of the *loading factor* is fulfilled if the value of the loading factor at the output is > 0.5. This indicates that the loading factor being tested is significant or has an influence on the grouped variables.

f. Interpreting the results of the factor analysis.

3. RESULT AND DISCUSSION

3.1. Results

3.1.1. KMO and Bartlett's Test

The first output of confirmatory factor analysis using IBM SPSS Statistics 21 is the result of the assumption test of the appropriateness of the research variables to fulfill standard factor analysis procedures. According to Table 4, the KMO value obtained is 0.750 which is in the range of values 0.7 ≤ KMO = 0.750 < 0.8. For the Kaiser assessment category, the value of KMO = 0.750 is included in the middling data criteria for factor analysis.

Table 4. KMO and Bartlett's test values for 12 variables

Kaiser-Meyer-Olkin Measures of Sampling Adequacy.		.750
Bartlett's Test of Sphericity	approx. Chi-Square	104.563
	df	66
	Sig.	002

Furthermore, the Bartlett's Test significance value = 0.002 < 0.05 which indicates a rejection of H_0 where the correlation matrix between the test variables is not an identity matrix so that principal component analysis can be completed. In other words, the variables that are the focus of the test are not correlated with one another in the population.

3.1.2. MSA

The second procedure is the MSA test (Measures of Sampling Adequacy) which aims (1) to measure the sampling adequacy of each variable to be predictable and further analyzed and (2) to select variables that have a low correlation index between variables to be reduced and re-analyzed on the variables that remaining. The MSA value in the SPSS output can be traced through the Anti-image Matrices table shown in Table 5.

Table 5. MSA values for 12 variables

Anti-image Matrices													
		VD1	VD2	VD3	VD4	VP1	VP2	VP3	VP4	VS1	VS2	VS3	VS4
Anti-image Correlation	VD1	.710 [a]	0.282	-0.079	-0.039	-0.005	0.159	0.004	-0.099	-0.236	-0.002	0.077	-0.021
	VD2	0.282	.537 [a]	-0.163	0.133	0.27	-0.046	-0.004	-0.079	0.115	0.104	0.252	-0.563
	VD3	-0.079	-0.163	.519 [a]	-0.276	0.044	-0.252	-0.158	-0.413	-0.1	-0.102	-0.129	0.12
	VD4	-0.039	0.133	-0.276	.489 [a]	-0.171	0.272	0.059	0.174	0.098	0.331	-0.05	-0.264
	VP1	-0.005	0.27	0.044	-0.171	.655 [a]	0.181	-0.005	-0.193	-0.132	0.2	0.235	-0.094
	VP2	0.159	-0.046	-0.252	0.272	0.181	.662 [a]	0.137	-0.09	0.027	0.163	-0.051	-0.017
	VP3	0.004	-0.004	-0.158	0.059	-0.005	0.137	.352 [a]	-0.246	0.017	0.164	0.214	-0.082
	VP4	-0.099	-0.079	-0.413	0.174	-0.193	-0.09	-0.246	.561 [a]	0.033	-0.12	-0.121	0.078
	VS1	-0.236	0.115	-0.1	0.098	-0.132	0.027	0.017	0.033	.631 [a]	0.052	-0.055	-0.132
	VS2	-0.002	0.104	-0.102	0.331	0.2	0.163	0.164	-0.12	0.052	. 510 [a]	0.067	-0.092
	VS3	0.077	0.252	-0.129	-0.05	0.235	-0.051	0.214	-0.121	-0.055	0.067	. 536 [a]	-0.247
	VS4	-0.021	-0.563	0.12	-0.264	-0.094	-0.017	-0.082	0.078	-0.132	-0.092	-0.247	.419 [a]

a. Measures of Sampling Adequacy (MSA)

Table 5 shows that of the 12 variables that will be further tested, there are 3 of them that have MSA <0.5, namely VD4, VP3 and VS4. Based on the MSA value criteria in Table 3, the three variables with an MSA value of <0.5 have a low correlation index between variables so they cannot be predicted and cannot be analyzed further. Thus the three variables VD4, VP3 and VS4 must be reduced from the model and retested on the other 9 variables.

The results of retesting on 9 variables increase the value of KMO which is presented in Table 6. Reducing inappropriate variables makes the latest KMO values fall into the good (meritorious) data category so that it can be said that the 9 surviving variables are more feasible for factor analysis.

Table 6. KMO and Bartlett's test values for 9 variables

Kaiser-Meyer-Olkin Measures of Sampling Adequacy.		.880
Bartlett's Test of Sphericity	approx. Chi-Square	64.332
	Df	36
	Sig.	003

Meanwhile, the MSA value for the 9 retested variables has a greater value of 0.5 (see Table 7) which indicates that the 9 variables can be predicted and analyzed further.

Table 7. MSA values for 9 variables

Anti-image Matrices										
		VD1	VD2	VD3	VP1	VP2	VP4	VS1	VS2	VS3
Anti-image Correlation	VD1	.516 [a]	0.027	0.011	-0.08	0.12	-0.072	-0.126	-0.096	0.033
	VD2	0.027	.617 [a]	-0.034	0.194	-0.259	0.1	0.137	-0.134	-0.12
	VD3	0.011	-0.034	.580 [a]	0.012	-0.106	-0.277	-0.056	0.038	-0.075
	VP1	-0.08	0.194	0.012	.664 [a]	0.212	0.004	0.106	0.06	-0.073
	VP2	0.12	-0.259	-0.106	0.212	.633 [a]	-0.117	-0.056	-0.1	0.131
	VP4	-0.072	0.1	-0.277	0.004	-0.117	.507 [a]	0.154	-0.102	-0.214
	VS1	-0.126	0.137	-0.056	0.106	-0.056	0.154	.588 [a]	0.229	-0.027
	VS2	-0.096	-0.134	0.038	0.06	-0.1	-0.102	0.229	.610 [a]	0.105
	VS3	0.033	-0.12	-0.075	-0.073	0.131	-0.214	-0.027	0.105	.570 [a]

a. Measures of Sampling Adequacy (MSA)

3.1.3. Factor Extraction

The purpose of factor extraction is to produce a number of factors according to the analysis criteria that are able to explain the correlation between the observed variables. In this study, factor extraction was achieved using principal component analysis which can be seen from the value of communalities in the SPSS calculation output. Factor criteria that are able to explain the variable must have a value of communalities > 0.5.

Table 8 shows the communalities value of 9 variables > 0.5 which can be interpreted that all the variables used can be explained by the factors that are formed and have a strong relationship with these factors. The greater the value of communalities, the better the factor analysis, this is because the greater the characteristics of the original variables that can be represented by the factors formed.

Table 8. Communality value

Communalities								
	Initial	Extraction		Initial	Extraction		Initial	Extraction
VD1	1.000	.778	VP1	1.000	.556	VS1	1.000	.751
VD2	1.000	.578	VP2	1.000	.593	VS2	1.000	.656
VD3	1.000	.575	VP4	1.000	.646	VS3	1.000	.528

Extraction Method: Principal Component Analysis.

For example, the closeness of the relationship between the variable VD1 and the factors formed is 0.778. This value implies that the contribution of VD1 to the factor formed is 77.8 % or the variable VD1 can explain the factor formed is 77.8%.

3.1.4. Number of Factors

The number of factors that can be formed can be seen from the eigenvalues. Eigenvalue is a value that shows how much influence a variable has on the formation of a factor's characteristics. In the SPSS calculation output, the eigenvalues of the factors formed are known from the total variance explained (see Table 9). The Total Variance Explained table shows the percentage of total variance that can be explained by the variety of factors formed. The accepted eigenvalue significance criterion is > 1, while the eigenvalue < 1 is not used because it has the ability to explain lower variance than the ability of the initial variable.

Table 9. Total variances explained

Components	Initial Eigenvalues			Extraction Sums of Squared Loadings			Rotation Sums of Squared Loadings		
	Total	% of Variances	cumulative %	Total	% of Variances	cumulative %	Total	% of Variances	cumulative %
1	1.89	36.996	36.996	1.89	36.996	36.996	1.723	35.145	35.145
2	1.429	19.882	56.878	1.429	19.882	56.878	1.446	32.071	67.216
3	1.206	13.402	70.280	1.206	13.402	70.280	1.338	30.87	70.303
4	1.035	11.502	81.782	1.035	11.502	81.782	1.053	11.479	81.782
5	0.892	6.906	88.688						
6	0.719	5.983	94.671						
7	0.658	3.309	97.980						
8	0.649	1.211	99.191						
9	0.523	.809	100						

Extraction Method: Principal Component Analysis.

Table 9 show that there are four factors that have an eigenvalue > 1, namely factor 1 with an eigenvalue of 1.89, factor 2 of 1.429, factor 3 of 1.206 and factor 4 of 1.035. The cumulative % column informs the cumulative percentage of variance that can be explained by factors. The amount of variance that can be explained by factor 1 is 36.996, the variance that can be explained by factors 1 and 2 is 56.878 and then the four factors are able to explain the total variance of 81.782. From these results it can be said that the four factors adequately represent the variance of the original variables.

The visual representation of the scree plot shows the number of factors formed. From Figure 2 it is known that the number of factors that must be maintained or stored in the main component is four. This is based on the extreme point of the curve line starting to slope shown in the fourth component.

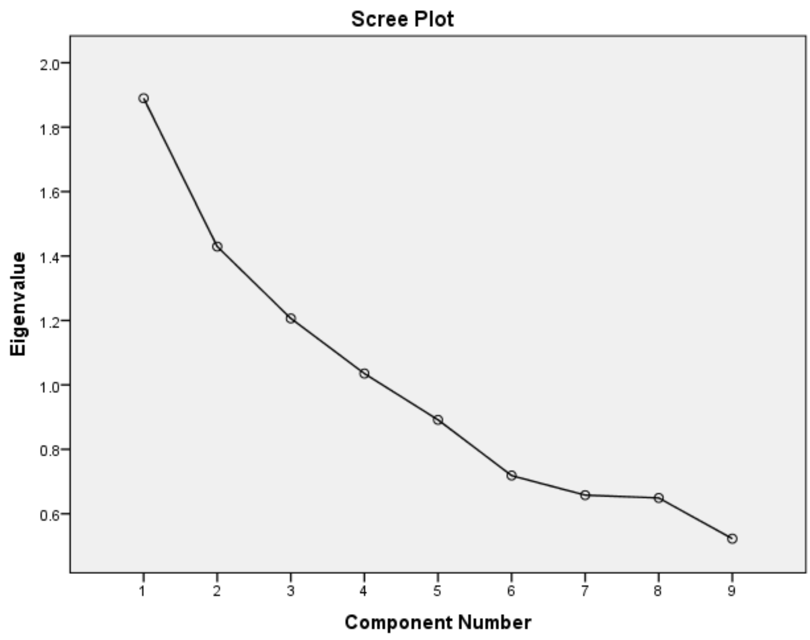


Figure 2. Scree plot number of factor components

3.1.5. Loading Factor

The factor that has been formed is called the loading factor whose value shows the correlation of each variable in the factor that is formed. Table 10 displays the four factors that are formed and produces a loading factor matrix. The values in the matrix are the correlation coefficients between the variables and the four factors. If you pay attention to Table 10, there are several variables that correlate with a factor but produce a correlation value with more than one coefficient interpretation. This makes it difficult to decide on the grouping of variables for each factor. For example, the VP2 variable correlates with factor 1 of 0.706 (strong correlation) and with factor 4 of 0.718 (strong correlation). In such conditions, it is difficult to decide whether the VP2 variable is included in the category of factor 1 or factor 4. The same situation occurs with the VS2 variable. Because each of the factors formed cannot be clearly interpreted as the position of the variable representation, it is necessary to do factor rotation.

Table 10. Matrix components

Component Matrix [a]				
	Components			
	1	2	3	4
VD1	-.265	.160	-.198	.802
VD2	.667	-.148	.018	-.100
VD3	.302	.578	-.351	.162
VP1	-.568	.305	.351	-.130
VP2	.706	-.138	.248	.718
VP4	.309	.729	-.053	.128
VS1	-.328	-.158	.706	.347
VS2	.507	-.030	.573	.262
VS3	-.039	.615	.172	-.345

Extraction Method: Principal Component Analysis.
a. 4 components extracted.

Factor rotation uses varimax method aims to maximize the variance of loading factor on each factor so that the original variable only has a strong correlation with one particular factor and a weak correlation with other factors. Table 11 show the result of factor rotation with varimax using SPSS which informs that each variable has a strong correlation with one factor. Because each factor has been able to explain the variance of the original variables correctly, thus the loading factor of the rotation results is in Table 11 used in the next analysis process. Table 11 show that the loading factor value for each variable is at a value of > 0.5 which indicates the factors formed are significant to the grouped variables.

Table 11. Varimax factor rotation

Rotated Component Matrix [a]				
	Components			
	1	2	3	4
VD1	.203	.061	.035	.856
VD2	.807	.033	.275	.180
VD3	.223	.707	.135	.083
VP1	.018	.053	.735	.110
VP2	.759	.114	.061	.017
VP4	.027	.753	.244	.137
VS1	.089	.052	.830	.224
VS2	.689	.026	.286	.314
VS3	.238	.696	.055	.337

Extraction Method: Principal Component Analysis.
Rotation Method: Varimax with Kaiser Normalization.
a. Rotation converged in 5 iterations.

3.1.6. Interpretation

The interpretation of the factors formed from a series of previous confirmatory factor analysis processes is as follows:

1) Factor 1 has a strong correlation with the variables VD2 (visual discrimination), VP2 (visual perception) and VS2 (visual analysis of shapes). The author named factor 1 as a strong factor in the visual thinking classification. The strong factor can explain the variance of data by 36.99 % with the largest loading factor value that appears in the VD2 indicator, namely 0.807.

2) Factor 2 has a strong correlation with the variables VD3 (visual discrimination), VP4 (visual perception) and VS3 (visual analysis of shapes). The author named factor 2 as the medium factor in the classification of visual thinking. The medium factor can explain the variance of the data by 19.88 % with the largest loading factor value that appears on the VP4 indicator, which is 0.753.

3) Factor 3 has a strong correlation with the variables VP1 (visual perception) and VS1 (visual analysis of shapes). The author named factor 3 as a sufficient factor in the visual thinking classification. The sufficient factor can explain the variance of the data by 13.40 % with the largest loading factor value that appears in the VS1 indicator, namely 0.830.

4) Factor 4 has a strong correlation with the variable VD1 (visual discrimination). The author named factor 4 as a sufficient factor in the visual thinking classification. The low factor can explain the variance of the data by 11.50 % with the largest loading factor value appearing on the VD1 indicator, namely 0.856.

Furthermore, to ensure that the factors formed have no further correlation between one another, it is necessary to trace the values in the component transformation matrix in

Table 12. Correlation values on the main diagonal for each factor lie in the range of values 0.8 to 0.9 which belong to in the category of very strong correlation. The implication of the correlation value is that the three factors that are formed can be said to be precise and have a unique closeness relationship.

Table 12. Component transformation matrix

Component Transformation Matrix				
Components	1	2	3	4
1	.854	.255	.441	-.107
2	-.330	.937	.108	047
3	.358	.238	.880	-.199
4	.183	032	-.137	.973

Extraction Method: Principal Component Analysis.

Rotation Method: Varimax with Kaiser Normalization.

3.2. Discussion

The confirmatory factor analysis procedure of the three aspects of the visual thinking classification which is detailed into 12 sub-variables produces the top four factors where the 9 variables in the four factors become predictors of the visual thinking classification aspects that develop in students after using a graphing quadratic worksheet. These four factors are hereinafter referred to as strong factors (factor 1), moderate (factor 2) and sufficient (factors 3 and 4). Table 11 informed that factor 1 of the visual thinking classification is represented by one variable each, namely VD2, VP2 and VS2. This data implies that VD2, VP2 and VS2 are included in the strong factors that contribute the most to the optimization of students' visual thinking abilities. It can also be interpreted that these three variables are the variables that spread the most in the data group with a percentage of variance of 36.99 %. In other words, students can best understand the concept of Quadratic Functions and are able to explore their knowledge and demonstrate a more dominant visual thinking performance through the three items VD2, VP2 and VS2 on the graphing quadratic worksheet. The same condition is shown in factor 2 of the visual thinking classification which is also represented by one variable each, namely VD3, VP4 and VS3. These three variables explain the variance of data by 19.88 %. This value indicates that the questions in the graphing quadratic worksheet represented by VD3, VP4 and VS3 can be digested and achieved by students, especially to stimulate their visual thinking skills. Furthermore, factor 3 of the visual thinking classification only correlates to the visual perception aspect represented by the VP1 variable and the visual analysis of shapes aspect represented by the VS1 variable with a percentage of variance of 13.40 %. This expresses that VP1 and VS1 still contribute to the classification of student visual thinking and can be used to explore this aspect. Finally, factor 4 of the visual thinking classification has only one correlation in the visual discrimination aspect represented by the VD1 variable with a variance percentage of 11.50 %. Based on the value of this variance, the item VD1 graphing quadratic worksheet is still in the fourth most spread variable category in the data group.

Recapitulation of the three aspects of visual thinking classification in the previous description which are represented by significant item variables in the graphing quadratic worksheet can be seen in Table 13. The interpretation contained of the numbers displayed in Table 13. is the visual thinking classification of the visual discrimination aspect can be improved through items VD2, VD3 and VD1 respectively based on the correlation values of the factors formed. Furthermore, the visual perception aspect can be optimized through items VP2, VP4 and VP1. Meanwhile, the visual analysis of shapes aspect can be supported by VS2, VS3 and VS1. The loading factor values of all the variables formed are significant and have a cumulative value of 81.78 % as shown in Table 9. This percentage value indicates that students' visual thinking abilities in the Quadratic Function material are at a fairly high level because it can be explained by a variety of variables of more than 80%.

Table 13. Visual thinking classification recapitulation

Classification of Visual Thinking	Sub Variable	Factor	Factor Loading	Information
The skill of visual discrimination (VD)	VD2	1	.807	Significant
	VD3	2	.707	Significant
	VD1	4	.856	Significant
The skill of visual perception (VP)	VP2	1	.759	Significant
	VP4	2	.753	Significant
	VP1	3	.735	Significant
The skill of visual analysis of shapes (VS)	VS2	1	.689	Significant
	VS3	2	.696	Significant
	VS1	3	.830	Significant

To explore which aspects of visual thinking classification are more developed in students after using a graphing quadratic worksheet can be seen in Table 14. In factors 1 and 2 all aspects of the visual thinking classification meet the significance criteria for loading factor. This indicates that the variables per aspect of the visual thinking classification developed in the graphing quadratic worksheet have a strong relationship with the visual thinking ability as a whole. So that it can be said that the aspect of visual thinking classification that develops in students after using a graphing quadratic worksheet is achieved in a balanced way. This is also emphasized by the cumulative variance value of factors 1 and 2 to be exact 56.88% of the total 81.78% for all factors. As explained by Deutsch and Beinker (2019) the greater the eigenvalue, the greater the contribution of the cumulative variance, the implication being that it is increasingly able to explain the variance of the original variables.

Table 14. Classification of visual thinking based on factor analysis

Factor	Sub Variable	Factor Loading	Information	Cumulative Variance
1	VD2	.807	Significant	36.996
	VP2	.759	Significant	
	VS2	.689	Significant	
2	VP4	.753	Significant	56.878
	VD3	.707	Significant	
	VS3	.696	Significant	
3	VS1	.830	Significant	70.280
	VP1	.735	Significant	
4	VD1	.856	Significant	81.782

A number of studies related to factor evaluation of an instrument either to review the validity of the instrument items or to investigate the extent to which the instrument can have an impact on users providing different types of knowledge and supporting the development of students' mathematical capabilities. The research results of tracing the most dominant visual thinking classification aspect emerged from students after using the graphing quadratic worksheet. This is consistent with a number of previous studies. The research conducted by Alsina et al. (2021) with a focus on analyzing learning assessment instruments to explore which mathematical processes among problem solving, reasoning and proof, communication, connection and representation are dominant and recessive in teaching practice. The results of the confirmatory analysis detected a change in scores on the mathematical connection process item. In addition, Semeraro et al. (2020) used confirmatory factor analysis in examining the role of cognitive factors, non-cognitive factors and the quality of student and teacher interactions, which are the best predictors of student achievement in mathematics. The results of this study revealed that cognitive ability was the best predictor of students' mathematics achievement. Furthermore, Wan et al. (2022) analyzed the construct validation of the project-based STEM learning experience scale from four dimensions namely scientific inquiry, technological application, engineering design and mathematical processing. The three studies have the same analysis pattern as this study, namely evaluating factors through testing the construct validity of the measuring instruments or instruments developed.

4. CONCLUSION

From the 12 visual thinking classification sub-variables, three variables were reduced that did not meet the MSA score, namely VD4, VP3 and VS4, resulting in 9 variables that met the criteria for factor analysis. The results of the confirmatory factor analysis formed four main factors with eigenvalues > 1 and explained a total variance of 81.78 % with the understanding that the four factors adequately represented the variance of the original variables. Factor 1 has a strong correlation with the variables VD2 (visual discrimination), VP2 (visual perception) and VS2 (visual analysis of shapes) which can explain the variance of the data by 36.99 % with the largest loading factor value appearing on the VD2 indicator, namely 0.807. This value denotes that the three items VD2, VP2 and VS2 on the graphing quadratic worksheet dominate the understanding of students in building an understanding of the concept of Quadratic Functions and stimulating their visual thinking abilities. Factor 2 has a strong correlation with the variables VD3 (visual discrimination), VP4 (visual perception) and VS3 (visual analysis of shapes) which can explain the variance of the data by 19.88 % with the largest loading factor value appearing on the VP4 indicator, namely 0.753. This value indicates that the question items in the graphing quadratic worksheet represented by VD3, VP4 and VS3 can be digested and achieved by students. In factors 1 and 2 all aspects of the visual thinking classification meet the significance criteria for loading factor. This gives an understanding that between the variables per aspect of the visual thinking classification developed in the graphing quadratic worksheet has a strong relationship with the ability of visual thinking as a whole. So that it can be said that the aspect of visual thinking classification that develops in students after using a graphing quadratic worksheet is achieved in a balanced way. This claim is highlighted by the cumulative variance value of factors 1 and 2 is 56.88% of the total 81.78% for all factors. Recommendations for further research are focused on tracing the relationship between latent variables that appear in this factor analysis using the Structural Equation Modeling method so that the direction of the relationship between the three latent variables as a whole can be known.

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