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GENERALIZATIONS AND ANALOGICAL REASONING OF JUNIOR HIGH SCHOOL VIEWED FROM BRUNER'S LEARNING THEORY

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ABSTRACT

Inductive reasoning has an important role in mathematics learning. It includes making generalizations and analogical reasoning. While a generalization explains the relationship between several concepts applied in more general situations, analogical reasoning compares two things. This research is qualitative and descriptive. It reviews and describes the mathematical reasoning abilities of junior high school students based on Bruner's learning theory. It was conducted at one of the junior high schools in Pekanbaru in the eighth grade in the 2022/2023 academic year, involving 70 students. The students were divided into three categories of prior mathematical knowledge: low, medium, and high. The instruments used to obtain data on how mathematical reasoning abilities relate to Bruner's learning theory in this study were (1) a test of mathematical reasoning abilities and 2) an interview guide. The results show that the average mathematical reasoning abilities of the eighth graders in this study were very high for the material on arithmetic sequences and series and low for the material on geometric sequences and series. However, the eight grade students' average generalizing and analogical reasoning abilities were quite good for both materials.

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1. INTRODUCTION

Mathematics is one of the subjects that are taught from an early age even up to the higher education level. This is because mathematics has an important role and benefits in everyday life (Angraini, 2019). Someone with an ability in mathematics can form a systematic mindset, reason, make conjectures, make decisions carefully, thoroughly, have curiosity, be creative, and be innovative (Erdem & Soylu, 2017; Habsah, 2017; Hidayat &

Husnussalam, 2019; Hidayat et al., 2018; Hutajulu et al., 2019; Sukirwan et al., 2018). In addition, mathematics is a tool used to support knowledge in the social, economic, and scientific fields. One example of the application of mathematics is used in personal or business financial management, such as calculating expenses, income, savings, interest, investments, and budget planning.

Mathematics has special characteristics, so mathematics learning needs to be handled specifically as well. One of the special characteristics of mathematics is its emphasis on deductive processes that require logical and axiomatic reasoning. Deductive thinking is essential in the context of mathematics for constructing proofs, establishing the truth of mathematical statements, and ensuring logical coherence. It forms the foundation of mathematical rigor and precision, enabling mathematicians to confidently explore, develop, and communicate mathematical ideas (Hidayat & Aripin, 2023; Ningsih et al., 2023). This also shows that mathematics emerges from thoughts related to ideas, processes, and reasoning (Bernard & Chotimah, 2018; Habsah, 2017; Herbert et al., 2015; Maharani, 2014; Nu'man, 2012). The process of thinking about mathematical ideas requires an understanding of problems related to the material being considered and the ability to reason.

The results of the 2018 Programme for International Student Assessment (PISA) indicate that the quality of education in Indonesia is rated the 75th out of 80 nations, with a decrease in PISA score in each subject area, including a drop from 386 to 379 in mathematics. In addition to these findings, the Trends in International Mathematics and Science Study (TIMSS) ranked Indonesia the 44th out of 49 nations. The 2015 TIMMS scores for mathematics achievement were 54% poor, 15% intermediate, and 6% high. The PISA and TIMMS findings indicate that mathematics education in Indonesia is of a very poor standard. The TIMSS results and decreased PISA scores in every subject area, including mathematics, indicate that Indonesian students have difficulty applying mathematical reasoning to solve complex mathematical problems. From this data, it can be concluded that there are serious problems in teaching and learning mathematics in Indonesia. Low mathematical reasoning ability can hinder students' development in understanding more complex mathematical concepts and can have a negative impact on their academic achievement in various fields. The PISA and TIMMS results are only one indicator that reflects the overall situation of mathematics education in Indonesia. However, there are still many factors that can affect students' mathematical abilities, such as curriculum, teaching methodology, teacher qualifications, student participation rates, socio-economic factors, and learning culture.

The reasoning is the main characteristic of mathematics that is inseparable from the activities of studying and solving mathematical problems (Jeannotte & Kieran, 2017; Norqvist et al., 2019). Reasoning abilities are critical to the understanding of mathematics. This is because mathematics is a science that has axiomatic deductive characteristics, which requires thinking and reasoning skills to understand it. The mathematical reasoning ability is the process of thinking mathematically in obtaining mathematical conclusions based on facts or available or relevant data, concepts, and methods (Lestari & Jailani, 2018; Mata-Pereira & da Ponte, 2017).

According to Jäder et al. (2017) and Hidayat et al. (2022) there are six general skills in reasoning, namely, (1) identifying similarities and differences, (2) problem-solving, (3) argumentation, (4) decision-making, (5) testing hypotheses and conducting scientific investigations, and (6) using logic and reason. The introducing students to reasoning has several advantages: (1) if students are given the opportunity to use their reasoning skills in making predictions based on their own experiences, they will remember it more easily; (2) if students are required to use their reasoning, it will encourage them to make conjectures;

and (3) it helps students understand the value of negative feedback in deciding an answer (Marasabessy, 2021; Mukuka, Balimuttajjo, et al., 2020; Mukuka, Mutarutinya, et al., 2020).

Bruner's theory is closely related to mathematics learning (Gading et al., 2017; Wen, 2018). Brunner (2019) stated that learning mathematics entails gaining an understanding of the mathematical ideas and structures included in the material being studied, as well as searching for correlations between these concepts and structures. Students must establish order by manipulating materials that correspond to intuitive regularities they already possess. Hence, they participate cognitively in the process of learning. Additionally, Bruner indicated that the greatest approach to learning is to comprehend ideas, meanings, and connections via an intuitive process.

Bruner proposed that learners construct their own knowledge by discovery learning (Joshi & Katiyar, 2021). In particular, in learning that encourages students to actively seek and acquire information from their experiences, students naturally offer outcomes to themselves and seek answers to issues via their own efforts, resulting in the production of meaningful knowledge (Inde et al., 2020; Rahmayanti, 2021; Tanjung et al., 2020). According to Brunner (2019), if the teacher allows students to discover a rule (including concepts, theories, and definitions) through examples that describe/represent the rules that are the source, the learning process will go smoothly and creatively; in other words, students will be led inductively to understand a general truth.

Inductive reasoning has an important role in the development of mathematics. By observing several cases, one can draw a conclusion as a generalization or abstraction of the cases. However, this conclusion is still provisional until it can be proven. If it has not been proven, then the pronouncement is only a conjecture. From this, it can be seen that the big role of inductive reasoning is to build mathematical knowledge because mathematical discoveries often occur through observations assisted by inductive reasoning.

Mathematics does have a deductive nature, as it relies on logical reasoning and the construction of deductive proofs to establish the truth of mathematical statements. However, used inductive reasoning in mathematics for several reasons: Inductive reasoning allows students to explore new concepts and patterns. Inductive reasoning helps students generate hypotheses that can guide their mathematical investigations. Inductive reasoning is also valuable in mathematics for identifying potential counterexamples. Inductive reasoning plays a role in developing mathematical intuition and creativity (English, 2013; Jablonski & Ludwig, 2022).

Inductive reasoning consists of making generalizations and analogical reasoning. A generalization is an explanation of the relationship between several concepts that are applied in more general situations. Conclusions drawn from inductive generalizations can take the forms of either rules or predictions based on those rules. Meanwhile, analogical reasoning compares two different things. An inductive analogy not only shows similarities between two different things; it also draws conclusions on the basis of those similarities. Analogies can help students understand a material through comparisons with other materials to look for similarities in nature between the materials being compared (English, 2013; Jablonski & Ludwig, 2022; Piaget, 1999).

Durak and Tutak (2019) and Sumartini (2015) explained that in mathematics, mathematical reasoning is the process of mathematically thinking in order to arrive at mathematical conclusions based on facts or accessible or relevant data, ideas, and procedures. Research results by Durak and Tutak (2019) and Sumartini (2015) shows that the increase in the mathematical reasoning abilities of students who receive problem-based learning is better than students who receive conventional learning. Gifted students have better reasoning skills in applying statistical concepts, interpreting data, or solving statistical problems compared to ordinary grade students. Reasoning is very important in mathematics learning because it constitutes a goal of mathematics learning, in addition to other goals related to understanding concepts that teachers already know, such as numbers, comparisons, geometry, and algebra.

Analogical reasoning is a cognitive process that involves identifying similarities and making connections between different situations or concepts. Studying analogical reasoning in junior high school students is important for fostering creativity and critical thinking. Analogical reasoning promotes creative thinking and the ability to see relationships and patterns between seemingly unrelated concepts. Exploring analogical reasoning in junior high school students can contribute to the development of their critical thinking skills and creativity.

Generalization refers to the ability to recognize and apply patterns or principles across different contexts. Understanding generalization in junior high school students is important for deepening conceptual understanding. Generalization requires students to identify commonalities and underlying principles among various examples or situations. By studying how students generalize mathematical concepts, educators can gain insights into the depth of their conceptual understanding.

This study aims to look at students' mathematical reasoning abilities, especially in terms of the generalizing and analogical reasoning abilities of the eight grade students in the materials on arithmetic and geometric sequences and series, in relation to Bruner's learning theory.

2. METHOD

This research is qualitative and descriptive in nature. This study describes the mathematical analogical reasoning and generalizing abilities of junior high school students based on Bruner's learning theory. The data analysis technique used is interpretative analysis. Interpretive analysis techniques involve interpretation and in-depth understanding of the data that has been collected. Researchers try to understand the context, meaning, and relationships that emerge from the data being analyzed. This understanding can be based on Bruner's learning theory and relevant frameworks, and supported by emerging findings from the data analysis.

This research was conducted on the eight grade students of Junior High Schools 34 Pekanbaru in the 2022/2023 academic year. There were a total of 70 the eight grade students at Junior High Schools 34 Pekanbaru, all of whom were taken as sample in this study, to obtain more in-depth information about the mathematical reasoning abilities of junior high school students for further research development. The students were divided into three categories of prior mathematical knowledge based on the scores gained from a test on previous materials, namely, low, medium, and high categories.

In this study, the mathematical analogical ability refers to the process of drawing a conclusion on the basis of similarities by comparing two different things. This conclusion can later be used to explain or as a basis for reasoning. The analogical ability was measured using these indicators (English, 2013): (1) the student could recognise patterns (from images or numbers) and (2) the student could ascertain how the visual patterns or numbers relate to one another.

The capacity to derive general inferences from the primary structures observed is in this study referred to as mathematical generalizing ability. Patterns, general principles, and specific examples are observed in accordance with some underlying rules. The indicators of the mathematical generalizing ability used in this study were as follow (English, 2013): (1) the student could produce general rules and patterns and (2) the student could use the generalizations that they had made to solve problems.

In this study, Bruner's theory of the learning process was implemented in the following operational steps: (1) determining learning objectives; (2) identifying the characteristics of the students (prior mathematical knowledge); and (3) conducting an assessment of student learning outcomes based on the theory of learning in three stages, namely, the enactive, iconic, and symbolic stages (Jablonski & Ludwig, 2022). The enactive stage is the learning stage where students are given the opportunity to manipulate concrete objects directly. The iconic stage is the learning stage where students manipulate concrete objects into images. The symbolic stage is the learning stage where students manipulate the images obtained from the previous stages into mathematical symbols.

The instruments used to obtain data on how mathematical reasoning abilities relate to Bruner's learning theory in this study were (1) a test of mathematical reasoning abilities and 2) an interview guide. Interviews were conducted directly between the researcher and the respondent, in which the researcher asked questions and received verbal answers from the respondent. Interviews were conducted in a semi-structured manner (some questions were predetermined but there was flexibility in asking additional questions) which were conducted after the administration of the questionnaire. The data obtained were calculated using statistical tests, and the results are to be explained in depth. The following are two of the questions in the mathematical reasoning abilities test taken by students (see Figure 1).

1. Theater A has a seating capacity of 720 seats. The relationship between movie theater A and 720 seats *is similar to* that between movie theater B and Y seats. The seating for the B movie theater is arranged from the front row to the back with many rows behind, more than 5 seats from the front row. If in the theater there are 20 rows of seats and in the front row there are 25 seats, the capacity of the theater is Y seats. Determine the value of Y.

2. The data obtained from daily observations of the height of a plant forms the following line:

Day	1	2	3	4	…	n
Height	$\frac{3}{2}$	2	$\frac{8}{3}$	$\frac{32}{9}$		

   a. Determine the plant height on day n.
   b. Determine the plant length on the 7th day.

Figure 1. the questions of the mathematical reasoning abilities

3. RESULT AND DISCUSSION

This study aims to look at the mathematical generalizing and analogical reasoning abilities of junior high school students for the materials on arithmetic and geometric sequences and series from the perspective of Bruner's learning theory. To gain information on the students' mathematical reasoning abilities for the materials, a test was administered with results provided in Table 1.

Table 1. Data on students' generalizing and analogical reasoning abilities

Descriptive statistics	Students
N	70
Means	70.67
sd	7.76
Max	90
Min	60

Table 1 show that average score of the generalizing and analogical reasoning abilities of the eight grade students of Junior High Schools 34 Pekanbaru was quite high, but none of the students achieved a score of 100. The highest score obtained was 90. Table 1 shows that overall, eighth grade students at SMP 34 Pekanbaru have good generalization and analogy thinking skills, as indicated by a high average score. This indicates that the majority of students have succeeded in applying mathematical thinking skills that involve generalizations and analogies in problem solving. Despite the high average scores, no student achieved a perfect score of 100 on the generalization and analogy thinking skills test. This shows that even though students have good abilities, there is still room for improvement and further development in their mathematical thinking skills. The highest score achieved by the students was 90, which shows that there are some students who have very good mathematical thinking skills and are close to a perfect score. However, there is potential for students to reach even higher levels of ability. The average score of generalizing and analogical reasoning abilities was then calculated based on the prior mathematical knowledge, which was measured from the score of a test on previous materials (see Table 2).

Table 2. Average generalizing and analogical reasoning abilities based on prior mathematical knowledge

Ability	N	Means
High	23	77
Medium	24	70
Low	23	65

Table 2 show that average generalizing and analogical reasoning abilities of students with high prior mathematical knowledge surpassed those of students with moderate and low prior mathematical knowledge by far. This shows that, descriptively, prior mathematical knowledge could also distinguish students' generalizing and analogical reasoning abilities. Meanwhile, from the questionnaire data obtained on the students' mathematical reasoning abilities, the following were found: 1) the students strongly agreed that they were able to solve the exercise questions on arithmetic sequences and series given by the teacher; 2) the students had a high level of confidence in learning the arithmetic sequences and series material; 3) the students fairly agreed that they were able to solve the exercise questions on geometric sequences and series given by the teacher; and 4) the students had a fair level of confidence in learning the geometric sequences and series material.

During the learning process on the arithmetic and geometric sequences and series materials, the teacher implemented Bruner's learning theory. Bruner is best known for his "discovery learning" concept. He said that mathematics learning will be successful if the teaching process is directed to the concepts and structures made in the subject being taught, in addition to the relationships between concepts and structures. He also explained several operational steps to implementing his theory of the learning process: (1) determining learning objectives; (2) identifying the students' characteristics (initial abilities), in addition to selecting the subject matter, determining the topics the students must learn inductively, developing the learning material in the form of inductive examples, illustrations, assignments, and so on for students to learn, and arranging lesson topics in the order from the simplest to the most complex, from the concrete to the abstract; and 3) conducting an assessment of the students' learning processes and outcomes.

In this study, the analogical reasoning ability was measured using three indicators: (1) the student could identify patterns (from images or numbers), (2) the student could ascertain the link between patterns or numbers; and (3) the student could estimate the rules behind the patterns. Meanwhile, the generalizing ability had two indicators: (1) the student could generate general rules and patterns and (2) the student could use the generalizations they had made to solve problems (Durak & Tutak, 2019; English, 2013; Jablonski & Ludwig, 2022; Sumartini, 2015). The following is the achievement of the generalizing and analogical reasoning abilities of the students for the arithmetic sequences and series material (see Table 3).

Table 3. Average scores of generalizing and analogical reasoning abilities of the arithmetic sequences and series material

Ability	N	Means
High	23	100
Medium	24	95
Low	23	90

The instrument used to measure the generalizing and analogical reasoning abilities of the students for the arithmetic sequences and series material consists of three questions. Almost all students, whether they had a low, medium, or high ability, were able to answer these questions. As explained in Brunner (2019) learning theory, if the teacher permits students to find a rule (including ideas, theories, and definitions) for themselves via examples that describe/represent the source rules, the learning process will go easily and creatively. In other words, students are guided inductively to comprehend a general truth. Bruner's learning theory has advantages in a number of ways: (1) It determines if learning is worthwhile using discovery learning; (2) The student's newfound information will be retained for a very long time and it will be easy to recall; (3) Problem-solving requires a great deal of discovery learning, which is preferred because it allows the student to display the information they have learned; (4) If generalizations are generated by the student for themselves rather than being supplied in final forms, transfer may be improved; (5) The use of discovery learning may help foster learning motivation; and (6) It enhances the student's logical thinking and capacity for independent thought.

Table 4. Average scores of generalizing and analogical reasoning abilities of the geometric sequences and series material

Ability	N	Means
High	23	54
Medium	24	45
Low	23	40

The instrument used to measure the students' generalizing and analogical reasoning abilities for the geometric sequences and series material consists of two questions. Almost all of the students, whether they had a low, medium, or high ability, were unable to answer these questions (see Table 4). The questionnaire data collected support this finding: a) the students fairly agreed that they were able to solve the exercise questions on geometric sequences and series given by the teacher; b) the students had a fair level of confidence in learning the geometric sequences and series material; c) the students were bored in solving the problems on geometric sequences and series; and d) the students had a hard time learning the geometric sequences and series material. The following are some examples of students' answers to the generalizing and analogical reasoning abilities test (see Figure 2).

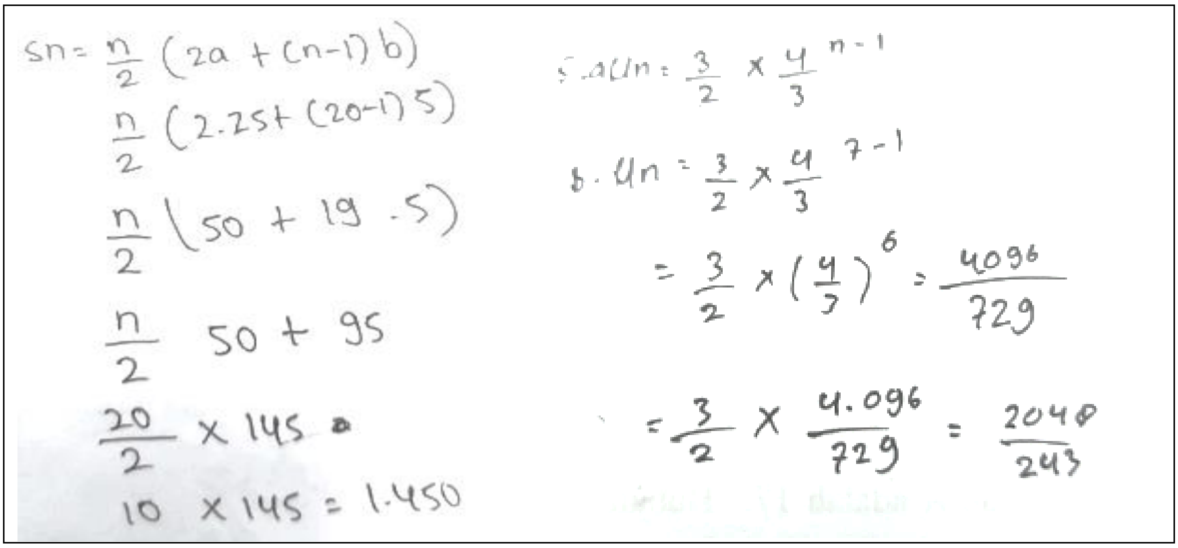

Figure 2. Students' answers to the mathematical reasoning abilities test

The following are the results of interviews between researchers and students regarding student test answers:

R : *How do you determine the value of Y.*

S : *To determine the value of Y, we need to find the pattern of the relationship between the number of seats and rows in theater A and theater B. In theater A, the seating capacity is 720 seats. In theater B, we know that there are 20 rows of seats, with 25 seats in the front row. However, it should be noted that many of the rows behind are 5 seats more than the front row.*

R : *What can you conclude from this information?*

S : *If each row of seats in theater B has 25 seats plus a few extra seats, then we can calculate the number of seats in theater B with the row formula and an arithmetic series.*

R : *Right. Now, we need to find out how many seats there are. Based on the information that there are 5 more seats in the back row than the front row, what can you conclude?*

S : *In this context, we know that there are 20 rows of seats as "n", with 25 seats in the front row as "a", the number of rows behind is 5 seats more than the front row as "b".*

R : *Very well. So, what is the value of Y based on your explanation?*

S : *Based on my calculations, the Y value is 1450 seats.*

R : *How would you solve problem number 2?*

S : *To solve this problem, I will look for the pattern of growth in plant height from day to day. In the data given, the plant height on the first day is 2. However, I need to find a general pattern to determine the plant height on the n day.*

R : *Good. What can you conclude from this data?*

S : *I see that in the data, the plant height for each day is not given. However, if I observe the height of the plants from the first to the second day, there is an increase. I can assume that there is a steady pattern of improvement every day.*

R : *Very well. With these assumptions, how can we determine the height of the plant on the n day?*

S : *If there is a steady pattern of increase in plant height each day, we can assume that plant height follows a geometric progression. In a geometric series, each term is divided by the previous term to give a fixed ratio.*

R : *Good. So, how can we determine the height of the plant on day n in this geometric series?*

S : *In a geometric series, we need to determine the growth ratio of the height of the plants. If the ratio is r, then the plant height on the nth day can be determined by the geometric sequence and series formulas.*

R : *Good. Now, let's apply the formula to this problem. The plant height on the first day is 2. Can you determine the plant height on the n day?*

S : *Yes, the plant height on day n is like the answer I wrote.*

R : *Very well. Now, let's look at the second question. How would you determine the length of the plant on day 7?*

S : *To determine the length of the plant on day 7, I need to determine the value of U7.*

R : *Good. Can you determine the value of U7?*

S : *Yes, if I replace the value of n with 7 in the geometric sequence and series formula, the answer is as I wrote it.*

Furthermore, interviews were conducted with students who had low prior mathematical knowledge:

R : *How do you feel about the material?*

S : *To be honest, I found it difficult to understand some of the concepts.*

R : *Can you explain in more detail about the difficulties you are facing?*

S : *I have difficulty understanding the meaning of the questions in the test. Sometimes the questions are complex and I am at a loss as to what to do. I am often confused about* *where to start. I sometimes use the wrong formula or forget the formula I am supposed to use.*

R : *I saw that. Is there anything else you think may have affected your test results?*

S : *Yes, I feel that I am often in a rush when working on questions. I want to solve it quickly, but sometimes I forget the formula. I also have difficulty distinguishing formulas in arithmetic sequences and series and geometric sequences and series. I often make mistakes in applying the right formula.*

R : *From our discussion, it seems that several factors such as understanding the question, understanding the approach, using the right formula, and speed of work influence your test results.*

S : *I think so.*

According to study by Guarino et al. (2021), it is possible to understand children's challenges with analogy problem solving by evaluating visual attention during analogy problem solving and a measure of inhibitory control, which is thought to be crucial to analogical thinking. There is a connection between visual attention, inhibitory control, and a number of behavioral performance indicators. According to Richland et al. (2006), the interplay between improvements in relational knowledge, the ability to integrate different relationships, and inhibitory control over featural distraction determines how analogical reasoning changes with age. Because a variety of abilities, such as originality, creativity, and inductive reasoning, are crucial for future scholastic success and professional success. Theoretically analogous thinking actually aids students in comprehending abstract notions that are then articulated or analogized to become tangible in studying mathematics. Additionally, by connecting concepts that were previously distinct into a single idea, pupils are able to learn new information or concepts. Following that, factors that must be taken into account when answering problems involving analogical reasoning include first ensuring that students have mastered any prerequisite knowledge or ideas. As a result, students can reduce conceptual blunders in their earlier learning and recognize concepts and problem-solving techniques that are included in the proper source problem that will aid in addressing the target problem.

Lee and Lee (2023) found potential connections between answers and their expected generalization reasoning challenges for pupils. The issue with generalization reasoning is that it frequently foresees problems with educational actions intended to support students' mathematical knowledge. Additionally, students overcome issues relating to their own self-efficacy and confidence by using generalization reasoning. Yao (2022) identified two categories of representation transformations that helped go from empirical to structural generalizations: a structurally beneficial treatment and a mathematically significant conversion. Mathematically significant conversions frequently result in generalizations that show why the generalizations are accurate, as opposed to structurally beneficial treatment.

Yao and Manouchehri (2019) state that to help learners become more proficient at constructing mathematical generalizations, it is vital to better understand the forms that the constructive process might take in various mathematical contexts. The study reported here aims to offer an empirically grounded theory of forms of generalization middle students made as they are engaged in explorations regarding geometric transformations within a dynamic geometry environment. Based on their sources, participants' statements about the properties of geometric transformations were categorized into four types: context-bounded properties, perception-based generalizations, process-based generalizations, and theory-based generalizations. Although these forms of generalizations are different in their construction process, with appropriate pedagogical support generalizations of the same type and different types of generalizations can build on each other. DGS mediated the construction of these forms of generalizations based on how learners used it.

According to Vamvakoussi (2017), it is important to take into account the usefulness of prior mathematical knowledge. Actually, critics of conceptual shift perspectives on learning claim that they overemphasize the negative consequences of existing knowledge while ignoring students' creative ideas that can serve as the foundation for additional learning. Teachers should frequently employ analogies as instructional mechanisms to teach concepts and procedures, selecting sources that are differentially generated to correspond to the analogy's content purpose (Richland & Begolli, 2016; Richland et al., 2004). Whether an analogy is used in response to a student's request for assistance depends on the source and target construction. Students frequently participate in the parts of the analogy that take the least amount of analogical thought, but teachers typically maintain control of each comparison by providing the majority of the comparison.

According to Costello (2017), case-based learning and other constructivist teaching strategies can help students gain the analytical and problem-solving abilities necessary for the modern workplace. According to Dias et al. (2020), the pedagogical approach could help children's intellectual growth by facilitating their learning through exploration, reaching all students, and encouraging the development of stochastic notions.

The inhibitory control factor that has been posited to contribute to the protracted development of analogical and also DGS mediated the construction of these forms of generalizations based on how learners used it. In this study the researcher also conducted interviews with several students regarding the results of the material test for geometric sequences and series, 3 students who were interviewed as representatives of each ability level stated that: 1) students had difficulty understanding the meaning of the questions; 2) students have difficulty determining a principle that will be applied to solve the problem; 3) students are mistaken in determining the right concept to solve the problem; 4) students are in a hurry to work on the questions; 5) students do not memorize formulas properly and correctly; 6) students cannot distinguish formulas in arithmetic sequences and series from geometric sequences or series and students are not careful in understanding the questions asked. Based on this, it is better for future research related to analogical reasoning and generalization to relate prior mathematical knowledge, inhibitory control, dynamic Geometry environment in improving analogical reasoning and generalization abilities.

These problems arose because solving problems on geometric sequences or series requires a higher level of thinking than solving problems on arithmetic sequences or series. Learning with Bruner's learning theory also has several weaknesses: 1) discovery learning requires high intelligence on the student’s part (if you are not smart enough, the learning will be less effective), and 2) theoretical learning takes time, and if it is not run in a guided or directed manner, it may result in chaos and uncertainty about the subject being studied.

The paragraphs provide a comprehensive discussion on various aspects related to analogical reasoning and generalization in mathematics education. Here are the key points derived from the provided information:

a) Importance of inhibitory control and visual attention: inhibitory control and visual attention play crucial roles in analogy problem solving. The ability to focus attention and control distractions is essential for successful analogical thinking.
b) Interplay between relational knowledge, integrative abilities, and inhibitory control: improvements in relational knowledge, the ability to integrate different relationships, and inhibitory control over distractions contribute to the development of analogical reasoning. These factors influence how analogical reasoning changes with age.

c) Benefits of analogical thinking in mathematics education: Analogical thinking helps students comprehend abstract notions by connecting concepts and making them tangible. It allows students to learn new information or concepts by connecting previously distinct ideas into a single idea.
d) Challenges in generalization reasoning: potential challenges in generalization reasoning, which may affect students' mathematical knowledge. Students can overcome these challenges by using generalization reasoning, which also contributes to their self-efficacy and confidence.
e) Forms of generalization in mathematics, four types of generalizations: context-bounded properties, perception-based generalizations, process-based generalizations, and theory-based generalizations. These forms of generalizations have different construction processes but can build on each other with appropriate pedagogical support.
f) Importance of prior mathematical knowledge: Prior mathematical knowledge is crucial when addressing problems involving analogical reasoning and generalization. It helps students reduce conceptual errors from earlier learning and recognize relevant concepts and problem-solving techniques.
g) Role of analogies and instructional strategies: Teachers can employ analogies as instructional mechanisms to teach concepts and procedures. Analogies should be selected based on their differential generation to correspond to the content purpose of the analogy.
h) Constructivist teaching strategies: the benefits of case-based learning and other constructivist teaching strategies in developing analytical and problem-solving abilities in students.
i) Challenges in solving geometric sequences and series problems: Based on interviews conducted with students, difficulties in understanding questions, determining applicable principles, and mistaking concepts were identified. Other challenges included rushing through questions, improper memorization of formulas, and difficulty distinguishing between arithmetic and geometric sequences or series. Considerations for future research: Future research should consider the relationship between prior mathematical knowledge, inhibitory control, and the dynamic Geometry environment to enhance analogical reasoning and generalization abilities.

In conclusion, the discussion highlights the importance of inhibitory control, visual attention, and prior mathematical knowledge in analogical reasoning and generalization. It emphasizes the role of analogies, instructional strategies, and constructivist approaches in facilitating students' mathematical learning. The challenges faced by students in solving geometric sequences and series problems provide insights for further research and instructional improvement.

4. CONCLUSION

The study's conclusion indicates that the average generalizing and analogical reasoning abilities of the eight grade students in this study for the arithmetic and geometric sequences and series materials as a whole was quite good. The average generalizing and analogical reasoning abilities of the students for the arithmetic sequences and series material alone was very high, but their abilities for the geometric sequences and series material were low.

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