Infinity Journal of Mathematics Education
Volume 12, No. 2, September 2023

p-ISSN 2089-6867
e–ISSN 2460-9285

https://doi.org/10.22460/infinity.v12i2.p377-392

PROVING GEOMETRY THEOREMS: STUDENT PROSPECTIVE TEACHERS' PERSEVERANCE AND MATHEMATICAL REASONING

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Article Info

Article history:

Received Jul 8, 2023
Revised Sep 7, 2023
Accepted Sep 14, 2023
Published Online Sep 22, 2023

Keywords:

Geometry theorem,
Mathematical reasoning,
Perseverance,
Proof

ABSTRACT

Proof of geometry is a topic that involves mathematical reasoning abilities and relates to perseverance involving hard work, the spirit of achievement, and self-confidence. The current important problem that occurs at this time is that students who are future teachers of mathematics still experience difficulties in compiling proofs, especially those who are not challenged to work hard. This qualitative research explores mathematics teacher candidates' reasoning abilities and perseverance in proving geometric theorems. Therefore, the research design used a case study. There were three participants in this study, and they were student prospective mathematics teachers' s taking geometry courses. Data were collected through working documents, open questionnaires, and semi-structured interviews and were analyzed using iterative techniques consisting of data condensation, data exposure, and verification. The study's results showed that students' prospective teachers did not prioritize proof in solving geometry problems, even though they worked hard to solve the problems independently until they were finished. The students' perseverance also impacts their mathematical reasoning in proving geometric theorems. Students with more hard work values tend to have more reasoning values. The results of this study have implications that there needs to be an effort from the teacher to get used to giving proof questions to support students' perseverance and mathematical reasoning abilities.

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How to Cite:

Aisyah, N., Susanti, E., Meryansumayeka, M., Siswono, T. Y. E., & Maat, S. M. (2023). Proving geometry theorems: Student prospective teachers' perseverance and mathematical reasoning. *Infinity*, *12*(2), 377-392.

1. INTRODUCTION

Geometry learning is an instructional activity that fosters reasoning skills through the proof process (Cai & Cirillo, 2014). In constructing formal proofs, students arrange informal language to formal language, understand mathematical definitions, understand and apply theorems, and make connections between mathematical objects (Corriveau, 2017; Di

Martino & Gregorio, 2019). Proof is at the heart of mathematical reasoning (Jeannotte & Kieran, 2017). The process of compiling evidence as a benchmark in mathematical reasoning (Lee et al., 2019). Proving often appears in geometry content standards and process standards in reasoning and proof (Fu et al., 2022). Proof in geometry is considered to be at a continuous high level of mathematical reasoning (Hanna, 2020). The geometry proof process is a manifestation of mathematical reasoning activity (Nathan et al., 2021).

Mathematical reasoning is one of the important topics in mathematics education (Hjelte et al., 2020). Mathematical reasoning has a significant role in students' thinking process (Hanna, 2020). When students can not develop their mathematical reasoning, they do not know the meaning of mathematical learning, it become a series of procedures and imitates examples (Rohaeti et al., 2020). Mathematical reasoning makes students familiar with mathematical symbols and objects, and students can use mathematics in various situations (Schliemann & Carraher, 2002). Various studies show that currently mathematics learning emphasizes mathematical reasoning. One of the PISA frameworks is mathematical reasoning at the secondary school level (OECD, 2019). The mathematics education curriculum in Japan emphasizes mathematical reasoning in grades 6-8 and also reasoning and proof are content standards in geometry (Fu et al., 2022). Mathematical reasoning is defined as a thought process in drawing conclusions based on facts or premises that are considered true with high deductive-logical quality (Nurjanah et al., 2021). Smit et al. (2022) revealed to support mathematical reasoning with three factors: learner characteristics (self-efficacy beliefs), learner achievement (in mathematical reasoning), and learning tasks, including feedback-supported classroom instruction.

At the higher education level, prospective mathematics teachers also need to have good reasoning skills. Reasoning in the proof process in geometry as a high level in education is important for prospective teachers in the introduction of the proof transition (Selden et al., 2014). Mathematical reasoning facilitates prospective mathematics teachers to understand formal mathematical language and axiomatic structures.

In carrying out the theorem proving process, prospective teachers often experience difficulties so that they are unable to complete the proof process (Güler, 2016). Students provide descriptions and solutions that they understand in solving problems. However, when they are not excited or lack confidence, the problem-solving process stops (Jäder et al., 2017). This shows that mathematical reasoning is influenced by various factors, not only from the cognitive aspect but also from the affective aspect of students during the learning process (Kurniansyah et al., 2022; Nathan et al., 2021). One of the essential to an individual's capacity to succeed at long-term goals and to persist in the face of challenges and obstacles is perseverance. DiNapoli and Miller (2022) define the construct of perseverance as "initiating and sustaining, and re-initiating and re-sustaining, in-the-moment productive struggle in the face of one or more obstacles, setbacks, or discouragements. Student perseverance is the willingness to engage in problem solving, even when facing problems and obstacles (Scherer & Gustafsson, 2015). Perseverance is another factor that affects students' cognitive abilities (Scherer & Gustafsson, 2015). The existence of perseverance makes students stay on task. Students with high perseverance are students who consistently choose to exert high effort, he remains focused on challenging tasks, works hard, and does not give up (Bettinger et al., 2018). Sengupta-Irving and Agarwal (2017) suggest that perseverance includes how students engage with the discipline in a particular context, and how they identify with the discipline more broadly. According to Barnes (2021), perseverance in mathematical reasoning is a hard effort to pursue a series of mathematical reasoning, regardless of the difficulty or duration in achieving success. As inevitable obstacles are approached during the process of learning mathematics with understanding, perseverance describes in-the-moment tenacity toward accomplishing a goal while also accounting for the malleability to alter a strategy when necessary (DiNapoli, 2023). In the context of problem-solving, perseverance is initiating and sustaining in-the-moment productive struggle in the face of mathematical obstacles, setbacks, or discouragements (DiNapoli, 2019). Yet, engaging in struggle can be grueling and is avoided by some students, and little is known about if and how student perseverance can improve over time (DiNapoli & Miller, 2022).

Previous research that examines the relationship between cognitive aspects of students in reasoning has been widely conducted (Öztürk & Kaplan, 2019). In addition, there are affective aspects that also affect students' reasoning ability (Furinghetti & Morselli, 2009; Nathan et al., 2021). When students solve non-routine problems there is a link between reasoning ability and student confidence (Jäder et al., 2017). DiNapoli and Miller (2022) describe the effect of scaffolding mathematics tasks on student perseverance. However, this research is presented quantitatively and is still general in nature. To see how student perseverance, in this case a prospective mathematics teacher, works in doing mathematical proofs, a qualitative study of several prospective mathematics teachers is needed. This study was conducted to consider the efforts that can be made in the future to help prospective teachers foster the ability of proof and their perseverance in doing mathematical proof. Therefore, this study aims to explore the mathematical reasoning and hard work of prospective teachers in carrying out the theorem proving process in geometry.

2. METHOD

2.1. Research Design

This study examines in depth perseverance and mathematical reasoning of prospective teachers in solving geometry proof problems. Therefore, the reseach use qualitative approach. This study only focused on three subjects who were considered to have characteristics in accordance with the research objectives, so the research design used a case study.

2.2. Research Subject

This research involved 72 undergraduate students of mathematics education program taking Geometry Course. They were grouped into three groups according to their reasoning ability (very good, moderate, and less), from each of which three students were randomly selected. Furthermore, based on recommendation of the lecturer, from these nine students five students were choosen, three of whom were willing to participate in this research. They were HI, MI, and BA.

2.3. Data Collection

Reasoning ability data were collected through tests and semi-structured interviews. The test consisted of three description questions about proving the theorem of geometric similarity and congruence. Data on the subject's perseverance was collected through an open-ended questionnaire in the form of a google form and supported by semi-structured interview data. The questionnaire consists of eight questions covering aspects of independence, priority, persistence to keep trying, and persistence in seeking information from various sources. To classify the test and questionnaire data, interviews were conducted with three selected subjects. To support the questionnaire data, at the time of the interview, students were faced with geometry problems of drawing, calculating, and proving types to find out the priority of the problems that students do, perseverance and students’ reasoning in the process of proving geometry theorems, while to support the test data, the interview focused on the flow of thinking subjects in solving geometry proof problems. all research instruments were compiled based on indicators of perseverance and mathematical reasoning.

According to Sengupta-Irving and Agarwal (2017), perseverance has indicators including: 1) students immediately solve the problems given independently correctly; 2) students prioritize working on problems first; 3) students work on difficult problems even though they try repeatedly; 4) students try to find information from various references in solving the problems given; 5) students work on all the problems given; 6) students try to find answers to difficult problems; 7) students do all the problems given on time; and 8) students try to study again when the learning results are not as expected.

Meanwhile, the mathematical reasoning abilities are measured by indicators from including: 1) Make relevant conjectures or assumptions; 2) Present a mathematical statement from a known set of mathematical objects; 3) Constructing evidence by providing logical reasoning; and 4) Draw conclusions from the proof results (Amir et al., 2018; Jeannotte & Kieran, 2017).

2.4. Analyzing of Data

The test, questionnaire, and interview data were analyzed using an iterative technique consisting of data condensation, data exposure, and verification (Miles et al., 2018). Solutions written by students were analyzed to analyse the occurrence of indicators of mathematical reasoning and perseverance.

3. RESULT AND DISCUSSION

3.1. Results

Students' perseverance in proving geometry theorems is explored through an open-ended questionnaire supported by interviews. There are 8 open-ended questions related to the value of hard work that must be answered by students. Based on the results of this questionnaire and interviews it appears that all students do not view theorem proving as a priority in solving geometry problems. Students avoid proof questions as long as there are non-proof questions. However, if the available problems are only a matter of proof, students still try to solve them thoroughly.

3.1.1. HI' Perseverance

HI is a student with non-dominant emergence of perseverance indicators. Among the seven indicators in the questionnaire, five indicators appeared on HI. For the first, and second indicators in the questionnaire, HI stated that he preferred to solve non-proving problems if given the choice between working on proof and non-proving problems. HI reasoned: "*because the proof problem is more complicated than the non-proof where the proof problem uses the theorem*". HI further stated: "*I will leave the difficult problem, then choose the problem that I consider easy, after the easy problems have all been answered then I will try to answer the difficult problem*". HI's reason for leaving this proof problem was reinforced during the interview: "*theorem proving problems are more complicated than non-proving problems because they require an understanding of the theorem and certain reasons. So, I prefer other problems*".

HI's perseverance began to appear in the fourth Indicator. HI will make various efforts to work on difficult proof problems if there are no other problem options. Hard work to work on this proof problem can be seen from HI's statement: "*I will look for references from journals, and also ask friends if the proof problem is difficult for me to do*". HI's effort to solve the proof problem was also revealed from the interview, "*If faced with difficult proof problems, I will try myself first, after that look for help through modules, YouTube, or other references and then ask friends*". This shows HI's perseverance to solve all problems independently based on his knowledge and refer to references if needed. HI will only ask a friend if it is no longer possible to do it independently. This statement also shows the emergence of indicators about trying to try repeatedly in working on difficult problems that did not appear in the questionnaire.

The next indicator relates to the time to solve the problem, HI prefers to be on time rather than solving the problems one by one completely. "*I prefer to complete the problems one by one*". HI further stated "*I start with the easier problems first, and if the time is not enough, I prefer to complete the problem I am working on even though there are problems that are not done*". HI's statement also shows the absence of indicators related to working on all problems.

If the learning results obtained were not satisfactory, HI stated that he would make more effort to study again: "*yes there is, a second chance to try as much as possible so that future test results are better*". HI further stated: "*I will first see which answers are correct and which are wrong, then see why they are wrong, and how to fix them*". This shows that HI has the hard work to always learn again from learning results that are not as expected.

3.1.2. MI’s Perseverance

MI is the student with the most dominant indicators of perseverance. Of the seven indicators, five indicators appeared in MI. The first indicator is prioritizing working on proof problems and difficult problems first rather than solving proof problems and working on difficult proof problems by trying them repeatedly. Related to this, MI gave a reason: "*Because for me, proof problems are quite difficult to solve. This can be seen when I worked on both types of problems. I am faster at working on non-proving problems than proving problems*". Even when during the interview, MI was shown examples of problems with various types, MI explicitly chose to solve drawing problems, calculation problems, and finally proof problems. MI added: "*I will prioritize non-proving problems even though the assessment weight is greater*". MI's last statement emphasizes that the problem of proof is not a priority for MI.

MI’s perseverance is seen in the second indicator, namely trying repeatedly if faced with difficult proof problems: "*I will try repeatedly until I get the right and correct answer because usually one problem is interrelated with the next problem*". Related to this, MI reasoned: "*Yes, I will try many times and try as much as possible to work on the difficult proof problems, I still try to find answers until the time runs out, for fear of not being accepted by the lecturer*." MI further stated: "*if I am faced with a proof problem, I will first try on my own and try to find help from various references*".

For the last indicator, if the results obtained are not optimal, MI stated that it is more challenging to complete the answer until it becomes new knowledge for me. However, this statement contradicts the interview where MI stated that "*I will repeat the missing answers to eliminate my curiosity*".

3.1.3. BA’s Perseverance

BA is categorized as a subject with the least occurrence of perseverance indicators. Of the seven existing indicators, only three indicators appeared on BA. The first indicator is prioritizing to work on proof problems including indicators that do not appear. In solving geometry problems, BA tends to avoid when faced with proof problems and choose to work on problems that are not proof first: "*The problem that I do first is not a problem of proof*". In the questionnaire BA did not give reasons for this statement, but in the interview, BA explicitly gave the following reasons.

Researcher	:	*for example, working on problems. There are three problems, number 1 is calculating, number 2 is drawing, number 3 is proving. Which problem will be done first?*
BA	:	*No. 2 ma'am.*
Researcher	:	*Okay, now I'm turning. Problem No. 1 is about proving, No. 2 is about drawing, No. 3 is about calculating. which one will be done first?*
BA	:	*Keep no. 2 ma'am.*

From the interview above, it can be seen that the indicator of prioritizing proof problems really does not appear on BA

BA is also categorized as a subject who tends to do all the problems given on time. if faced with several problems related to proof, BA will try to solve one problem, but if the expected solution has not been found, BA will move to another problem: "*I chose to be on time even though I did not finish because I was committed to the lecturer*". BA emphasized this statement in the interview: "*I will try to solve all problems even though I can only do a little and not completely*". BA's statement in the questionnaire and interview also shows LI's lack of effort to work on difficult proof problems repeatedly. This statement also implicitly shows the emergence of the indicator of taking the initiative to work on all proof problems. According to BA: "*If I solve problem after problem, there is definitely not enough time because the proof problem is difficult. So just do as much as I can and move on to other problems*".

BA's perseverance can be seen in the third indicator, namely trying to find information from various references independently in working on proving problems: "*I try to find various references from online media myself if I am still confused, I will ask my friends*". During the interview BA stated: "*If I have difficulty solving the problem, I will usually ask a friend*". When asked why he did not try various references, BA stated: "*Sometimes it's still difficult too*". This statement shows that BA's perseverance to solve this proof problem did not appear.

When learning results were not as expected and the references used were not supportive, BA tried to re-learn the lesson: "*Yes, I will repeat the lesson again until I think I understand*". Furthermore, in the interview, BA stated that she would ask her friends and discuss things that she did not understand in order to better understand the material. BA's statement shows the emergence of the last indicator, namely trying to re-learn, when learning results are not in line with expectations.

Students' mathematical reasoning in proving geometry theorems is explored through tests given after geometry lectures. There are three proof-oriented test items that must be answered by students. These test questions were analyzed by focusing on the emergence of indicators of mathematical reasoning value in students as described in Table 1.

Table 1. Students' mathematical reasoning score in proving geometry theorem

Indicator	HI	MI	BA
1. Make relevant conjectures or assumptions	√	×	×
2. Present a mathematical statement from a known set of mathematical objects	√	√	√
3. Constructing evidence by providing logical reasoning	√	√	×
4. Draw conclusions from the proof results	√	√	×

Description: √ = appears × = does not appear

3.1.4. HI Mathematical Reasoning Score

From the three proof problems given, HI answered all the questions and it was seen that all indicators of mathematical reasoning value appeared in the completion of the problem.

The first indicator is making conjectures and assumptions implicitly seen from the drawings made by HI. HI makes the assumption that equal angles are angle A and angle D, angle C and angle F, while comparable sides are AC and DF (see Figure 1).

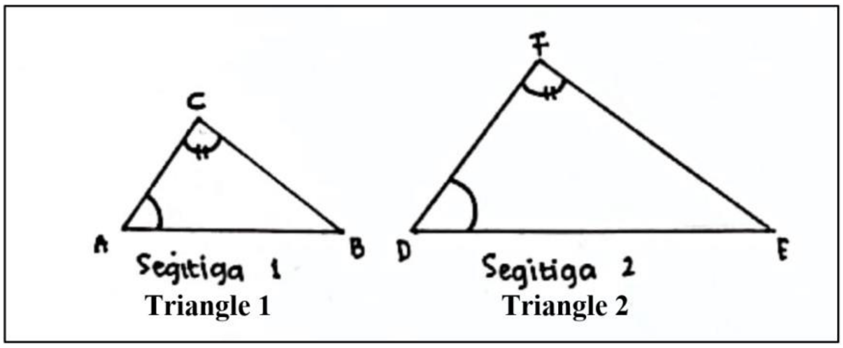

Figure 1. HI assumption

HI stated "*Based on the problem, I suppose there is a first triangle and a second triangle, the first and second triangles are the same shape but different in size*". this statement implicitly shows that HI assumes the two triangles that have been drawn are congruent.

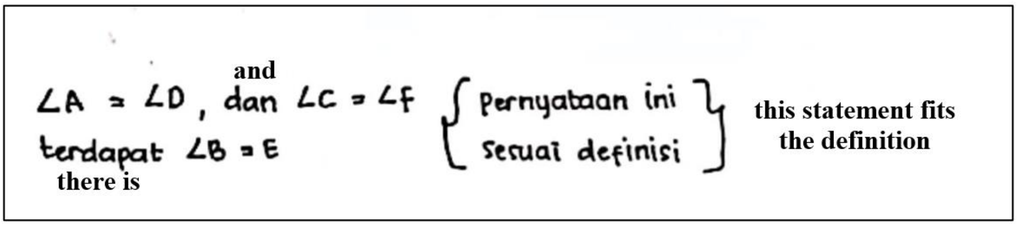

Figure 2. HI assumption

From the answer (see Figure 2), HI made the statement that ∠B=∠E based on the assumption that ∠A=∠D; ∠C=∠F. This answer is reinforced by HI's statement that

"*∠A=∠D; ∠C=∠F are known to be equal in magnitude, so ∠B=∠E must also be equal in magnitude. This is in accordance with the definition of a congruent triangle*".

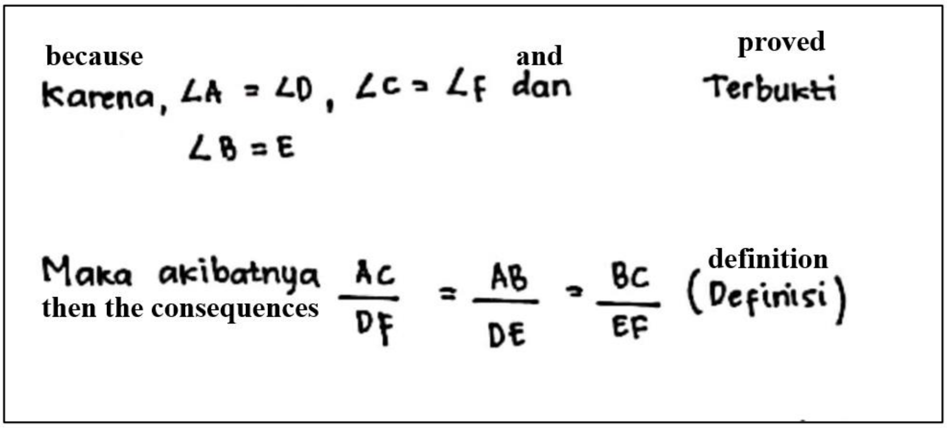

Figure 3. HI assumption

The indicator of compiling proofs by providing relevant reasons can be seen from each step of the proof done by HI. For example, for the statement if ∠A=∠D and ∠C=∠F then ∠B=∠E is supported by reasoning in accordance with the definition. Furthermore, HI also gave reasons in accordance with the definition when stating that all the corresponding angles in the two triangles have the same angle magnitude and are known to be the same. $\frac{AC}{DF}$ consequently $\frac{AC}{DF}=\frac{AB}{DE}=\frac{BC}{EF}$. This is in accordance with the definition of a congruent triangle (see Figure 3).

This answer is reinforced by HI's statement "*A=∠D; ∠C=∠F; ∠B=∠E is proven based on the definition of a congruent triangle, so the side of the triangle is also in accordance with the definition of a congruent triangle.* $\frac{AC}{DF}=\frac{AB}{DE}=\frac{BC}{EF}$ *this is also in accordance with the definition of a congruent triangle*".

This answer is reinforced by HI's statement "*A=∠D; ∠C=∠F; ∠B=∠E is proven based on the definition of a congruent triangle, so the side of the triangle is also in accordance with the definition of a congruent triangle.* $\frac{AC}{DF}=\frac{AB}{DE}=\frac{BC}{EF}$ *this is also in accordance with the definition of a congruent triangle*".

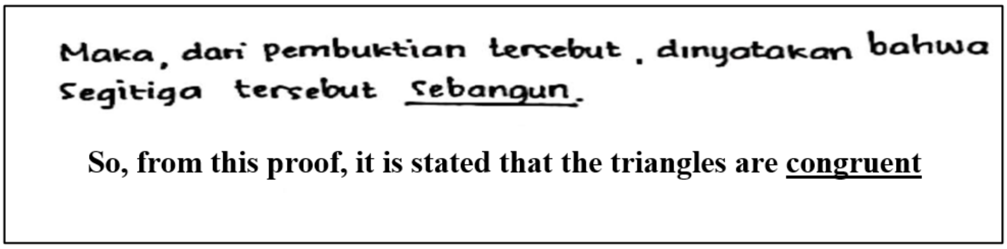

Figure 4. HI assumption

The indicator of making conclusions from the results of the proof is seen from the conclusions made by HI at the end of the proof (see Figure 4). HI stated "*I conclude that the two triangles are congruent because the conditions match the definition*". From this HI statement HI implicitly concluded that if two angles of two triangles are equal, then the third angle is also equal and the two triangles must be congruent.

3.1.5. MI Mathematical Reasoning Score

MI is a student with imperfect indicator emergence. From the test results MI has wrong assumptions and does not make reasons for each step of the proof. The information in the problem is that there is quadrilateral ABCD where AB // CD. MI assumes ABCD is a parallelogram with AB = CD, this assumption contradicts the problem. Assuming that ABCD is a perpendicular, then the conjecture that two triangles are congruent is wrong (see Figure 5).

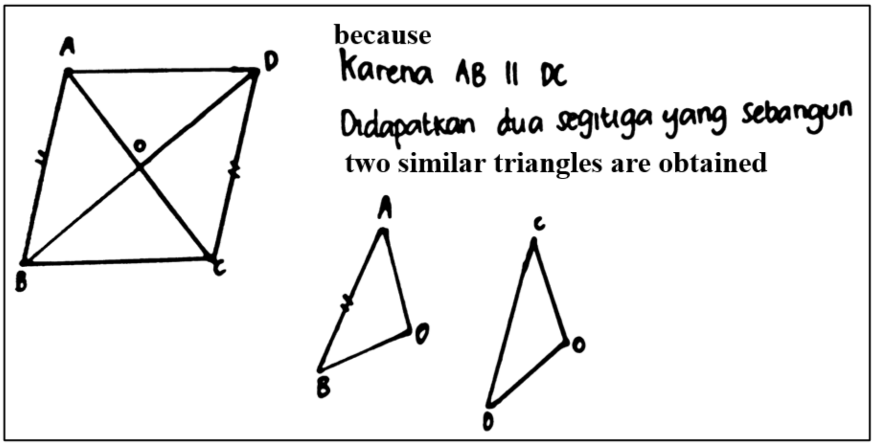

Figure 5. MI assumption

This incorrect assumption is also supported from the interview "*Because it is known that the quadrilateral ABCD and AB//DC. AC and BD intersect into the diagonal of the quadrilateral. The intersection produces point O and to make it easier sir. I focus on the line AB//DC, so ABCD is a parallelogram*".

From the statement, it can be seen that the assumptions made by MI are too specific. However, this specific assumption did not affect the subsequent proof process. For example, MI has made and presented mathematical statements correctly from a collection of known objects correctly even though each proof is not accompanied by reasons (see Figure 6).

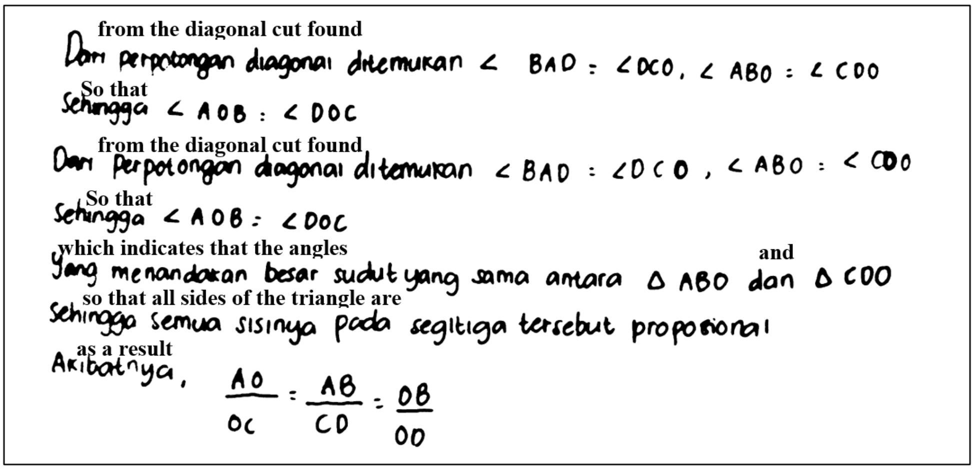

Figure 6. MI assumption

Based on the statement (see Figure 7), it can be seen that the reasoning indicators in MI have appeared even though they are well organized. The emergence of reasoning indicators in MI is supported by the following interview.

P : *Explain the process of proving* $\frac{AO}{OC} = \frac{OB}{OC}$

MI : *I made a parallelogram then its diagonal. You can see that ∠AOB is opposite ∠DOC so the two angles are equal. Then there is ∠BAO and ∠OCD because AB//DC then it is opposite so ∠BAO = ∠DCO*

P : *So why does ∠ABO = ∠CDO?*

MI : *Same thing sir, like ∠BAO = ∠DCO because it's the same thing.*

P : *How do you continue with the side comparisons?*

MI : *That's because the three sides are known to be equal and based on the definition of a congruent triangle, the side ratio is the same. It is proven that* $\frac{AO}{OC} = \frac{OB}{OC}$

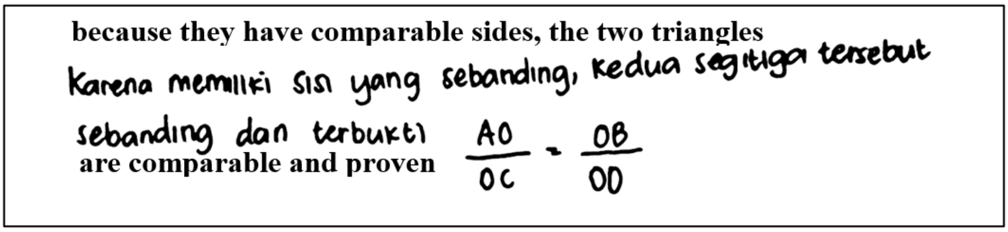

Figure 7. MI assumption

3.1.6. BA Mathematical Reasoning Score

BA is a student with only one appearance of mathematical reasoning indicators. BA's test results have made incorrect assumptions, did not organize the proof perfectly, and inappropriate conclusions. In the problem, it is only known that an angle of one triangle is congruent to the corresponding angle of another triangle. BA assumed that the two triangles were congruent, so the assumption that the two triangles were congruent was wrong (see Figure 8).

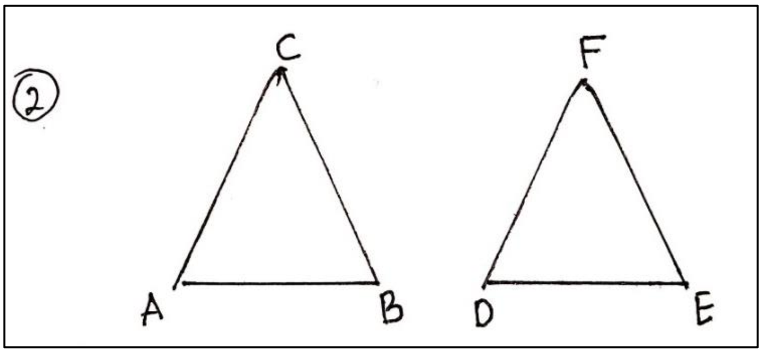

Figure 8. BA assumption

This inaccurate assumption is also supported from the interview "*Suppose there are two congruent triangles, meaning they have the same length. So, I made triangle ABC and* *triangle DEF*". from the statement it can be seen that the assumptions made by BA are too specific.

This specific assumption influenced the subsequent proof process. For example, BA made mathematical statements correctly. However, the proof process is not accompanied by logical argumentation. so that BA directly leads to the conclusion made, namely that the two triangles are congruent (see Figure 9).

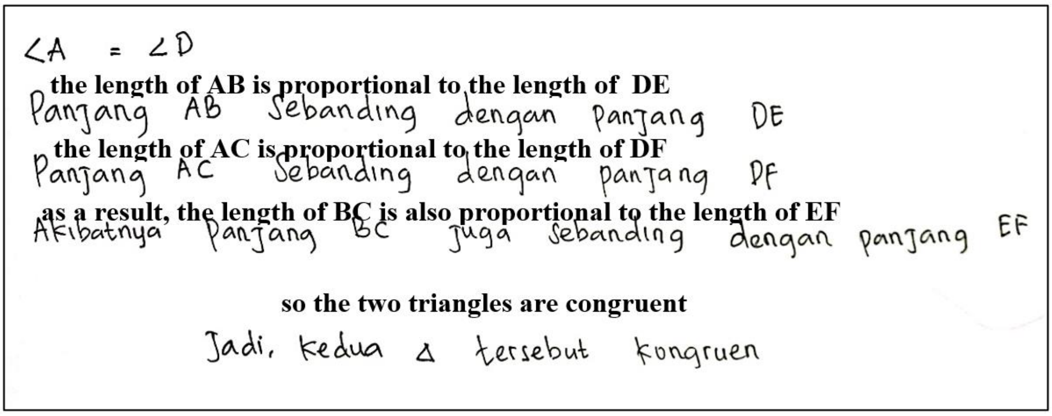

Figure 9. BA assumption

This is supported by the following interview with BA.

P : *Next, please tell me how you proved the theorem!*

BA : *I made triangle ABC and triangle DEF. so the corresponding angles are ∠A=∠D; ∠C=∠F; and angle ∠B=∠E. meaning the two triangles are congruent. The corresponding angles are equal and the corresponding sides are equal in length, meaning that ∠A=∠D is equal and the triangles are exactly AC/DF=AB/DE=BC/EF so it is proven that the triangles are congruent.*

3.2. Discussion

The results showed that the subject's perseverance in proving geometry theorems was still not explicitly visible. The lowest indicator occurrence is prioritizing to work on the proof problem first. All research subjects did not make proof problems a priority in solving Geometry problems. When the subjects were given problems in the form of proof and non-proving, all subjects chose to work on non-proving problems first before solving the proof problem. The most common reason stated was that the proof problem was a difficult problem to solve. The results of the study are in line with the results of previous studies which state that difficulties in solving proof problems are not only faced by students (Cyr, 2011), but also prospective teachers face the same thing (Güler, 2016; Ozdemir & Ovez, 2012).

The highest occurrence of the indicator is trying to learn again, when the learning results are not as expected. Efforts to continue learning can help students to understand the mathematical concepts needed for proof. By learning these concepts in depth, students will have a strong foundation to start the proof (Nadlifah & Prabawanto, 2017). Continuous learning efforts can also help students to develop the logical and deductive thinking skills needed for mathematical proofs. Students need to develop the logical and deductive thinking skills needed for mathematical proof because these skills are the basis for mathematical proof ability. Mathematical proof is the process of showing the truth of a mathematical statement or concept using valid mathematical principles. To be able to do proof well, students must have the ability to think logically and deductively, namely by using valid reasons and connecting mathematical concepts logically (Durand-Guerrier et al., 2012).

One interesting finding from this study is on the indicator of doing all the problems given on time. Subjects with dominant indicators of perseverance tend to choose to solve some problems completely rather than doing all the problems but not completely. This is in line with the results of research by Bettinger et al. (2018) which shows that students who have a high value of hard work will focus and not give up in working on difficult problems given.

In relation to the subject's reasoning value, the one indicator that appeared the least was making relevant conjectures or assumptions. Two subjects were unable to make assumptions correctly, related to the similarity and congruence of flat buildings. In proving the congruence of flat buildings, both subjects made assumptions that were too specific so that the resulting proof could not be generalized. This condition is in line with a case study conducted by Oflaz et al. (2016) which shows one of the proof schemes carried out by prospective teachers is to use inductive proof where prospective teachers provide examples to help them at the beginning of the proof process to then complete the proof and make generalizations.

One of the most dominant indicators that appeared in the subject's mathematical reasoning score was presenting mathematical statements from a collection of known mathematical objects. The three subjects have made mathematical statements appropriately from relevant concepts although there are still subjects who do not make reasons for each proof. This shows that the subject's procedural ability is good so that it supports the complex proof process (Firdausy et al., 2021; Jeannotte & Kieran, 2017).

Subjects with the most dominant perseverance showed systematic deductive thinking ability. In preparing the proof of a theorem, the subject used logical assumptions. While the subject with the least value of hard work tends to use specific assumptions in preparing the proof. The generalization produced by the subject is obtained from special cases instead of generalizing in general. The findings of this study are in line with Barnes (2021) that students' hard work value affects their mathematical reasoning ability so that efforts are needed to increase students' hard work value.

Given the importance of the role of perseverance on the process of student reasoning in the proof of geometry theorem, then perseverance should be considered by lecturers to be one of the important aspects in the learning process of geometry, especially affective aspects. One of the efforts that can be done to develop perseverance of this student by prioritizing the provision of proof questions on each topic of geometry. Accustoming students to compile this proof will provide benefits not only to the ability of proof and reasoning, but also to the development of perseverance of students who will have an impact on the ability of students to solve difficult mathematical problems (Barnes, 2021; DiNapoli & Miller, 2022; Sengupta-Irving & Agarwal, 2017).

4. CONCLUSION

The perseverance of students is not explicitly seen when solving proof problems. The most dominant indicator that appears when trying to solve proof problems is trying to learn again, when the learning results are not in line with expectations. In relation to students' mathematical reasoning ability, the most dominant indicator that appears is presenting mathematical statements from a collection of known mathematical objects. The results showed that students who have perseverance tend to have high mathematical reasoning values and vice versa. Apart from the view of all subjects that the proof problem is a difficult problem to solve, there is a tendency for subjects who have perseverance to solve several problems completely rather than doing all the problems but not completely.

The results of this study have implications for teachers to often provide proof problems to train students' mathematical reasoning skills as well as familiarize students with perseverance to be able to solve these proof problems. In addition, the research results can be a reference for policy makers in formulating the school curriculum.

Further research needs to be done to explore the learning environment that can support students' mathematical reasoning skills and perseverance in solving proof problems. This study has limitations that only involve a subject of 3 student teachers so that the results of the study in the form of a description of perseverance and mathematical reasoning ability of prospective teachers in proving geometry theorem is not strong enough to be generalized.

ACKNOWLEDGEMENTS

The authors would like to thank the students involved in this study, Diki Suryanto for assisting in data collection, and Prof. Dr. Rully Charitas Indra Prahmana for providing assistance in writing scientific articles.

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