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ELEMENTARY SCHOOL STUDENTS' LAYERS OF UNDERSTANDING IN SOLVING LITERACY PROBLEMS BASED ON SIDOARJO CONTEXT

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ABSTRACT

Elementary school students' difficulties in solving literacy problems are caused by students' lack of understanding of mathematical concepts. Using a specific regional context in literacy problems is essential to analyze layers of understanding elementary school more deeply. Based on Pirie-Kieren's theory, the study aims to analyze layers of understanding elementary school students in solving literacy problems in the context of Sidoarjo. The theory consists of eight understanding layers: primitive knowing, image making, image having, property noticing, formalising, observing, structuring, and inventing. A qualitative approach with a case study type was used as the research method. The research participants were 26 fifth-grade elementary school students. The determination of the research subjects was done by purposive technique. In this, the emphasis of determining the subject is taken on three categories of understanding by having one student each in the low, medium, and high categories. The results show that the layers of understanding of elementary school students are at the highest, namely observing. In addition, literacy problems in Sidoarjo contexts can overstimulate students' understanding. The context of learning should be used in Sidoarjo. Future research recommendations suggested using the regional context as a background for literacy problems in learning or problem-solving; this is needed to explore and develop layers of understanding to a higher level.

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1. INTRODUCTION

Math literacy is one of the indicators of the International Education Standards and the objectives of the PISA (Programme for International Student Assessment) (Holenstein et al., 2021). Math literacy is seen as the ability of students to understand and solve mathematical problems in various situations in everyday life (OECD, 2019). Meanwhile, many experts elaborate on the importance of math literacy. Wagner (2011), Holenstein et al.

(2021), and Kusuma et al. (2022) explained the importance of math literacy as at the core of mathematics learning so that students not only understand mathematical ideas, but are also helped to identify the role of mathematics in everyday life. In addition, Genc and Erbas (2019) explained that mathematics literacy plays an important role in the learning process in future education so that students are helped to understand the use of mathematics and make the right decisions. Stacey and Turner (2015) explained the importance of mathematics literacy for students to have adequate formulating, applying, and interpreting. In more depth, formulating so that students can recognize and identify problems that apply mathematics for solutions. Applying so that students can apply the concepts, facts, procedures, and mathematical reasoning that are formed to be solved mathematically. Interpreting so that students can interpret final results, solutions, or mathematical conclusions in real life.

Based on the survey results from PISA 2018, students' performance in solving math literacy problems in Indonesia was still relatively low. Indonesia ranks 74^{th} out of 79 countries with low literacy skills; it can be seen that the score obtained is 371 points, which is still far from the international PISA average score of 487 points (OECD, 2019). The difficulties are caused by a lack of ability to understand mathematical concepts and not using real contexts to solve mathematical problems (Celik, 2019; Kolar & Hodnik, 2021). So, it can be said that students' understanding of math literacy is still low. Students' inability to solve literacy problems shows their understanding of the material has not been conveyed properly.

One of the important factors that support math literacy problem-solving is understanding. This is because students will think systematically to solve problems in everyday life (Gülkılık et al., 2015; Posicelskaya et al., 2023). Baumert et al. (2012) also explained that understanding is part of mathematics literacy, so it also has an important role in solving literacy problems. Furthermore, in NCTM (2000), it can be interpreted that understanding mathematics will actively build new meaningful knowledge from experience and previous knowledge in various contexts. The presence of context supports students in developing their understanding of math literacy.

Using real contexts is necessary to facilitate students in solving math literacy problems. Real contexts are important in connecting math literacy problems with real-life experiences (Berisha & Bytyqi, 2020). Clair (2018) suggested that using real context should be near to the students. Yaro et al. (2020) explained that the presence of real contexts can attract students' attention to the thinking activities involved. Using real contexts in math literacy can provide an understanding of concepts and the true meaning of mathematical concepts (Umbara & Suryadi, 2019). Strong understanding is important and influences the quality of students' understanding at the education level (Rahayuningsih et al., 2022). Understanding is a comprehensive, dynamic, continuous, and non-linear growth process (Pirie & Kieren, 1994).

Layers of understanding are essential to measure students' level of understanding in learning and thinking (Suindayati et al., 2019). Pirie-Kieren theory is considered appropriate for analyzing students' math literacy understanding because it has a well-structured perspective on mathematical understanding (Martin, 2008). The Pirie-Kieren theory has eight layers to measure students' level of understanding in learning and thinking. The eight layers are primitive knowing, image making, image having, property noticing, formalising, observing, structuring, and inventising. The specialty of Pirie-Kieren's theory is the existence of layers of understanding and several constituent components in each layer and the existence of folding back, which is the activity of returning to the deepest layer when experiencing an obstacle in solving problems in the outer layers.

However, there is still a lack of evidence regarding how layers of understanding are formed in solving math literacy using real-life contexts. This is because several researchers have only provided research results in the last decade regarding using real contexts to solve math literacy problems (Efriani et al., 2019; Kholid & Nissa, 2022; Susanta et al., 2023). Efriani et al. (2019) used the context of sailing at the Asian Games as a setting for math literacy problems. This is in line with research from Kholid and Nissa (2022) using real contexts related to Papuan cities to solve mathematical literacy problems. The research of Susanta et al. (2023) uses the local context of Bengkulu city, which is familiar with everyday life. From some of the research, some previous studies have used real-life contexts, such as sports and local contexts. Real context is also important because it can provide strategies for students to solve math literacy.

The results of previous research and literature review show that real context is needed to solve math literacy and improve understanding. In this, the improvement in understanding means that students more easily understand the meaning of math literacy or more quickly and successfully complete math literacy that has a real context (Kolar & Hodnik, 2021; Rojas & Benakli, 2020; Rusdiana et al., 2023; Susanta et al., 2023). Rojas and Benakli (2020) said that math literacy problems with various real contexts can help students to build students' understanding. The results of research conducted by Kolar and Hodnik (2021) explain that students are more successful in solving math literacy problems using real contexts. Susanta et al. (2023) found the real context in the form of traditional houses and tourism in the student's area, making it easier for students to understand and solve the problem. Rusdiana et al. (2023) stated that solving literacy problems that have a real context by presenting problems set in the place where students live has made students solve problems faster.

Finally, presenting real context in solving math literacy problems for students is needed to help students understand the problem so that they can successfully solve it. This study uses the Sidoarjo context to ground the math literacy problem. The Sidoarjo context is used because it is the area where students live, so it is known as a real-world environment. In addition, Sidoarjo is a city rich in culture and local wisdom (Manoy & Purbaningrum, 2021). Considering the importance of Sidoarjo's context to mathematical literacy in improving understanding. Furthermore, no existing study has been able to answer how layers of student understanding are used in solving math literacy problems. This study focuses on the layers of student understanding in solving math literacy problems using the Sidoarjo context. Therefore, this study was designed to explore the characteristics of the level of understanding used by elementary schools in solving math literacy problems based on the context of Sidoarjo city.

2. METHOD

The research method used is qualitative with a case study approach. Qualitative research uses descriptive data from spoken or written words from people and actors that can be observed. Meanwhile, the case study approach is an investigative strategy that explores a program, process, event, and activity in depth (Creswell, 2012). This study aims to describe elementary school understanding in solving math literacy problems in the Sidoarjo context in terms of the Pirie-Kieren theory.

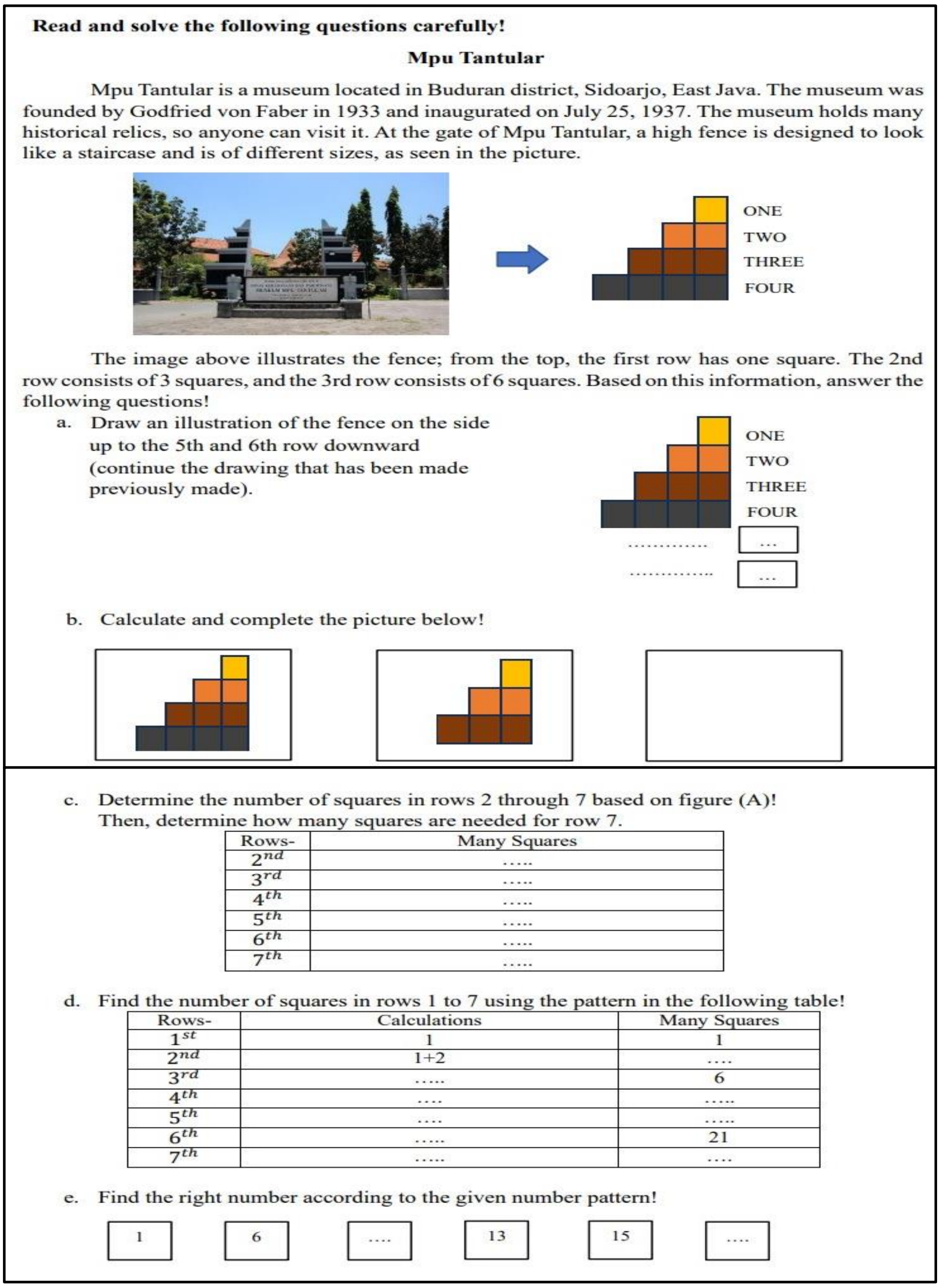
The 26 students in this study were in the fifth grade of elementary school at SD Muhammadiyah 2 Sidoarjo. The students involved were determined through a purposive sampling technique with certain considerations (Creswell, 2012). In this study, fifth-grade students were selected because of empirical evidence from previous studies, e.g., Chen et al. (2015), Brezovszky et al. (2019), and Apsari et al. (2020) have confirmed that fifth-grade students have an understanding in solving number pattern problems. In addition, they have received instruction about number patterns, so they are considered to have sufficient knowledge.

The selection of subjects to represent the exploration of students' understanding layers was carried out in several stages. First, students' work was assessed using an assessment rubric based on the characteristics of activity in solving literacy problems synthesized from Rahayuningsih et al. (2022) and Susanta et al. (2023), namely structured solution strategy, students' language use and symbolic operation skills, and representation. The assessment rubric and an explanation of each characteristic, indicator, and scale used are presented in Table 1. Second, grouping students into three categories of understanding in solving math literacy problems adapted from Rahayuningsih et al. (2022): low, medium, and high. Third, selecting one subject each in the low, medium, and high categories to represent students' understanding layers in solving math literacy. The selection of one subject was based on Creswell (2012) opinion that qualitative research that is narrative in nature that describes a person's experience can be conducted on only one individual. In this study, in order for the subject chosen to represent students' understanding layers in each category, one subject was chosen by adjusting the characteristics of each category. In the low category, the student chosen as the subject is the one with the lowest low score. In the medium category, the student chosen as the subject is the one with the middle score. In the high category, the student chosen as the subject is the one with the highest score.

Table 1. Assessment rubric

Characteristics	**Indicator**	**Rating Scale**		
		1	**2**	**3**
Structured solution strategy	Applying mathematical understanding in various ways, strategies through various procedures, and solving in a structured manner.	Not applying mathematical understanding and strategies in solving problems.	Applying mathematical understanding and strategies with various procedures. However, did not solve in a structured manner.	Applying mathematical understanding in a variety of ways and strategies in procedures, as well as solvingin a structured manner.
Language use and symbolic operation skills	Using the concept of number patterns correctly, using numbers/mathematical symbols correctly, and answering coherently and clearly.	Not using the concept of number patterns, and not answering clearly and coherently.	Using the concept of appropriate number patterns and correct use of numbers or symbols. However, answering the problem was not clear.	Using the right number pattern concept, using numbers or symbols correctly, and answering clearly and coherently.
Representation	Connecting various forms of representation in the form of images or numbers to solve problems	Not connecting the representations in the form of pictures or numbers.	Connecting representations in the form of pictures or numbers, but inaccurately.	Connecting representations in the form of pictures or numbers appropriately.

The research instruments used tests and semi-structured interviews. The test consisted of one math literacy problem used to find students' understanding characteristics according to Pirie-Kieren's theory. The question is set in the context of Sidoarjo. The context in question is a historical place, namely Mpu Tantular. Furthermore, semi-structured interviews were conducted on each subject as additional information for researchers. Meanwhile, the interview guideline question items regarding: (1) How did you solve this problem?; (2) What were you thinking about in this step of the solution?; (3) Explain the relationship between the current step of the solution and the previous step of the solution!". Therefore, some questions in the interview are carried out according to the guidelines, and questions will be developed according to the results of solving each subject's math literacy problem. The test instrument that has been translated into English is shown in Figure 1. Meanwhile, the indicators of layers of understanding of the Pirie-Kieren theory are presented in Table 2.



Read and solve the following questions carefully!

Mpu Tantular

Mpu Tantular is a museum located in Buduran district, Sidoarjo, East Java. The museum was founded by Godfried von Faber in 1933 and inaugurated on July 25, 1937. The museum holds many historical relics, so anyone can visit it. At the gate of Mpu Tantular, a high fence is designed to look like a staircase and is of different sizes, as seen in the picture.

The image above illustrates the fence; from the top, the first row has one square. The 2nd row consists of 3 squares, and the 3rd row consists of 6 squares. Based on this information, answer the following questions!

a. Draw an illustration of the fence on the side up to the 5th and 6th row downward (continue the drawing that has been made previously made).

b. Calculate and complete the picture below!

c. Determine the number of squares in rows 2 through 7 based on figure (A)! Then, determine how many squares are needed for row 7.

Rows-	Many Squares
2^{nd}	.....
3^{rd}	.....
4^{th}	.....
5^{th}	.....
6^{th}	.....
7^{th}	.....

d. Find the number of squares in rows 1 to 7 using the pattern in the following table!

Rows-	Calculations	Many Squares
1^{st}	1	1
2^{nd}	1+2	....
3^{rd}	.....	6
4^{th}	....	.....
5^{th}	....	.....
6^{th}	.....	21
7^{th}	.....	....

e. Find the right number according to the given number pattern!

1	6	....	13	15	....

Figure 1. Test instrument

Table 2. Indicators of layers of understanding of the Pirie-Kieren theory

Layers of Understanding	Indicators
Primitive Knowing	- Have a visual understanding of number patterns. - Identifying initial understanding of number patterns.
Image Making	- Have an idea to solve the problem. - Connect pattern items using numbers and pictures. - Create the next fence illustration correctly.
Image Having	- Continues drawing the pattern without engaging in specific activities such as providing symbols. - Relate the concept of pattern visually appropriately.
Property Noticing	- Determine the characteristics of a number pattern. - Demonstrates linkage of pattern items.
Formalising	- State the concept of number according to its properties. - Discover the concept of number patterns found by themselves to solve the given problem.
Observing	- Noted the previous pattern to solve the given problem. - Use patterns to form special equations that represent existing patterns.
Structuring	- Capture a pattern from the observations that have been made. - Find the concept of a number pattern and relate it to solve the given problem.
Inventising	- Create new concepts as a result of understanding number patterns. - Have a complete and structured understanding of patterns, and solve the given problems correctly.

Data collection techniques used subject selection based on predetermined math literacy activity components indicators and semi-structured interviews. The stages and techniques of analysis refer to the procedure by Creswell (2012), which includes: 1) reducing data, namely selecting data by examining the results of the subject's work, which will be analyzed according to Pirie-Kieren's theory of understanding, 2) exposure or presentation of data, namely by reading all the data presented based on the analysis of each indicator, and 3) drawing conclusions, namely concluding the data that has been obtained by describing students' abilities related to literacy problems based on mathematical understanding with Pirie-Kieren theory.

3. RESULT AND DISCUSSION

3.1. Results

The results of selecting three subjects from 26 students in each category of low, medium, and high understanding in solving math literacy problems are presented in Table 3.

Table 3. Subject selection results

Categories	Score intervals	Total	Subjects	Score
Low	$0 \leq x < 60$	7 student	S1 (33)	33
Medium	$60 \leq x < 80$	9 student	S2 (78)	78
High	$80 \leq x < 100$	10 student	S3 (89)	89

Description:
x = student understanding score in solving math literacy on a scale of 0 -100
S1-S3 = subject 1 to subject 3

Each student in the category was coded as S1 (subject 1), S2 (subject 2), and S3 (subject 3). S1 is the student who has the lowest score in the low category, S2 is the student who has the middle score in the medium category, and S3 is the student who has the highest score in the high category (see Table 3). Meanwhile, Table 4 describes the synthesis of selected subjects S1, S2, and S3 in each category.

Table 4. Subject description

Subject	Category	Description
S1	Low	Continue the drawing appropriately in accordance with the information presented in the problem. However, the pattern was abstractly not found and didn't solve the structured problem.
S2	Medium	Continue the drawing appropriately according to the information presented in the problem and find patterns abstractly. However, it didn't solve the structured problem.
S3	High	Correctly continue the drawing according to the information presented in the problem, find patterns abstractly, and solve structured problems.

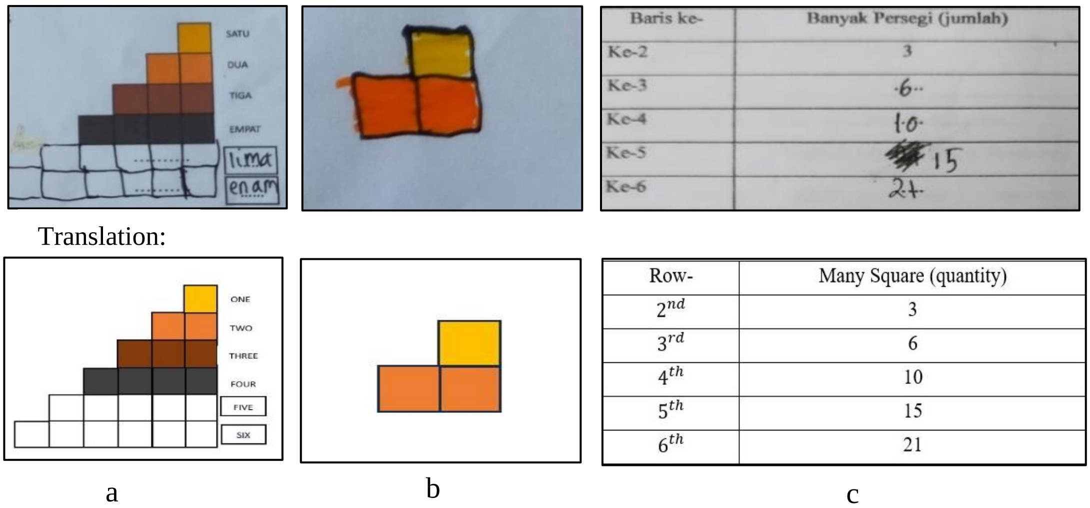


Figure 2. Work results from S1: a) image making, b) image having, c) property noticing

S1's work in Figure 2 shows an understanding of the image-making layer. Figure 2a, shows that S1 is able to continue the picture correctly. In addition, the interview results also explained that S1 had an idea to solve the problem. To convince his answer, S1 experienced folding back to primitive knowing, as the interview results showed that S1 understands the enlargement pattern.

In Figure 2b, S1 is in the image having layer because the subject can create and determine the image correctly without certain activities such as giving symbols of number. To convince his answer, S1 again experienced folding back, it can be seen from the interview that the subject understands the shrinking pattern. Figure 2c shows S1's understanding is at the property noticing layer because he can determine the characteristics of the pattern in the problem. Before going to that layer, S1 first experienced folding back to the image making layer which was used as a reference to answer the question. To show S1's understanding in solving literacy problems, the following results of the completion activity interview are presented.

Researcher : *How did you solve the test questions?*

S1 : *For problem (a), I looked at the picture in the reading of the problem because rows 1, 2, 3, and 4 were known. So, I thought and answered for the next drawing by drawing row 5 with 5 squares and row 6 with 6 squares (**image making**). In addition, I also noticed that the image was getting bigger and bigger **(primitive knowing).** For problem (b), I solved it by looking at the first box (the leftmost corner), where there are 4 rows, and then the second box has 3 rows. Then, I thought, "to fill the next box, there must be 2 rows" **(image having).***

Researcher : *Why do you think the next picture should have 2 rows?*

S1 : *Because there is a jump from 4 rows to 3 rows, then, from 3 rows, it should be 2 rows. Based on that, I thought and drew 2 lines **(image having).** In addition, I saw that the problem had a decreasing pattern **(primitive knowing).** For the answer (c), I initially answered by calculating the image like a question (a) folding back, and then I found that there was a pattern that was getting bigger, namely +3, +4, +5, +6 **(property noticing)**.*

Based on the work of the test questions and interviews with S1, the layer of mathematical understanding can be presented as in Figure 3.

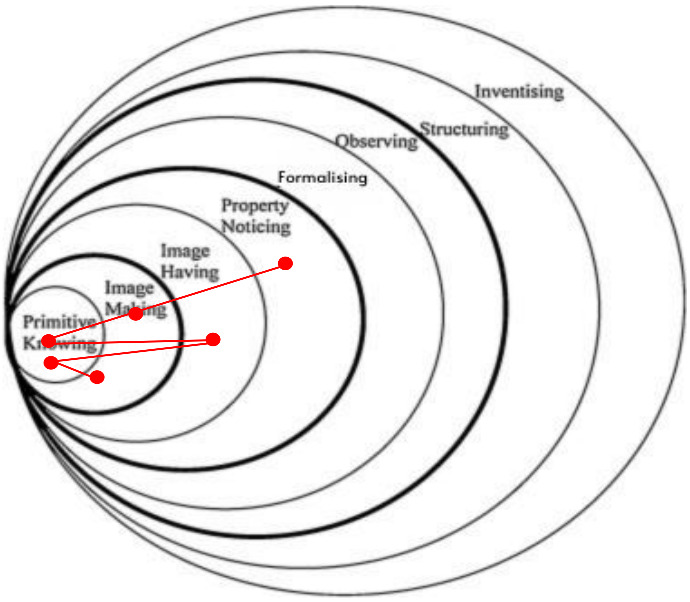


Figure 3. Layers of mathematical understanding S1

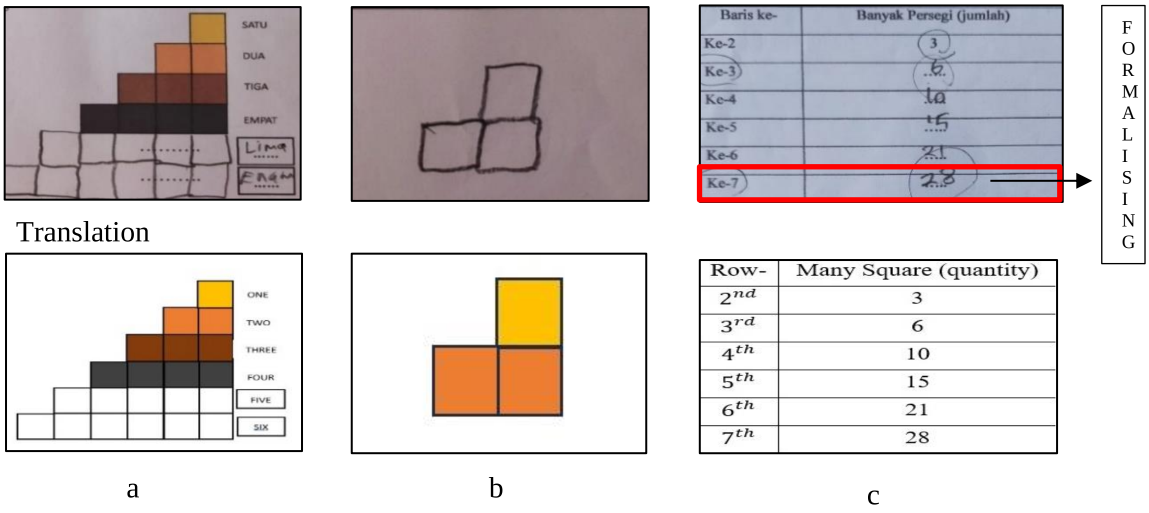


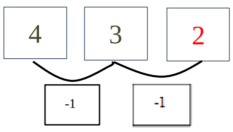
Figure 4. Work result S2: a) image making, b) image having, c) property noticing and formalising

Based on the interview results, S2 is at the primitive knowing layer. This is because S2 has an understanding of number patterns. In Figure 4a, S2 is in the image making layer because he has an idea and is able to make the next fence illustration correctly. Figure 4b shows that S2 is in the image having a layer because it can continue its drawing without engaging in certain activities. However, S2 experienced folding back to the primitive knowing layer to check his answer.

In Figure 4c, S2 is in the property noticing layer because it is able to determine the characteristics of a pattern in the problem, namely +3,+4,+5,+6,+7. Based on the interview results, S2 experienced folding back to the image making layer to convince his answer. Furthermore, S2 is at the Formalising layer because it is able to find patterns abstractly so that it can answer the question on line 7 correctly. Although, S2 has experienced folding back to the property noticing layer, which is used as a reference in answering the question. To show S2's understanding in solving literacy problems, the following results of the completion activity interview are presented.

Researcher : *How did you solve the test question?*

S2 : *For question (a), it has an enlarged pattern. Therefore, the next picture should be more than the previous one* ***(primitive knowing)****. Also, since the last row is 4, the next questions are 5 and 6. Then, I drew 5 squares for row 5 and 6 squares for row 6* ***(image making)****. For problem (b), the first box (left corner) has four rows, and the second box has 3 rows. Therefore, the third box should have two rows* ***(image having)****. Where the initial image has four rows, the further to the right, it looks reduced. Then, for the next image, it should be further reduced* ***(primitive knowing)****, which can be seen as below picture:*



Researcher : *How did you solve a problem (c)?*

S2 : *For question c, I initially looked at figure (a). Then, the question is row 3, and I added rows 1+2+3 for 6 squares. After I counted up to the 6th row, I found the jumps, and the pattern is getting bigger, as seen in the results, which*

		continue to increase. From row 2 to 6 there are jumps which are +3, +4, +5, +6 ***(property noticing)****.*
Researcher	:	*Row 7 does not have a picture like in problem (a). How did you do it?*
S2	:	*I worked it out from the sum of the squares of row 6, and there are 21. Then, finding the 7th row requires 7 squares. Therefore, I added 7. So, 21+7= 28. Also, there is a jump for each row. The question is the 7th row, so jump (+7)* ***(formalising)****.*

Based on the work of the test questions and interviews with S2, the layer of mathematical understanding can be presented as in Figure 5.

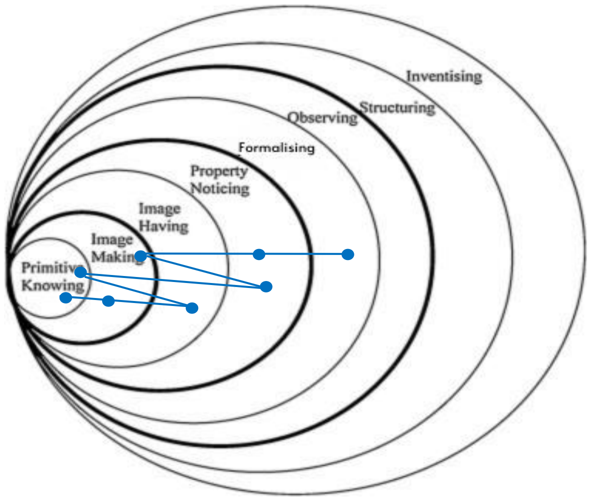
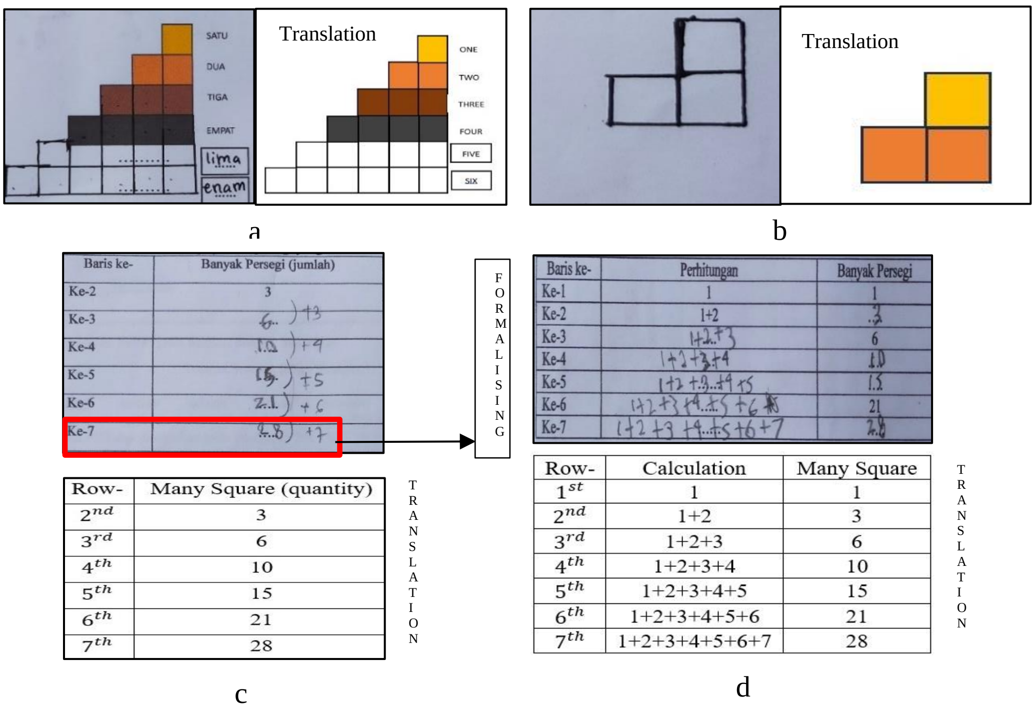


Figure 5. Layers of mathematical understanding S2



Row-	Many Square (quantity)
2^{nd}	3
3^{rd}	6
4^{th}	10
5^{th}	15
6^{th}	21
7^{th}	28

Row-	Calculation	Many Square
1^{st}	1	1
2^{nd}	1+2	3
3^{rd}	1+2+3	6
4^{th}	1+2+3+4	10
5^{th}	1+2+3+4+5	15
6^{th}	1+2+3+4+5+6	21
7^{th}	1+2+3+4+5+6+7	28

Figure 6. Work result S3: a) image making, b) image having, c) property noticing and formalising, d) observing

Based on the interview results, S3 is in the primitive knowing layer because S3 already understands the concept of number patterns. In Figure 6a, S3 is in the image-making layer. This can be seen when S3 can draw correctly and understands the enlarged pattern. In Figure 6b, S3 is in the image having a layer, where the subject can answer and associate patterns visually appropriately. When going to the image having layer, S3 experiences folding back to the primitive knowing layer because the subject understands that in the problem, there is a shrinking pattern, where each box will decrease by one row.

When going to the property noticing layer, S3 experiences folding back to the image-making layer, which is the first step to completing and convincing the answer. Thus, when in the property, noticing layer S3 can determine the characteristics of the pattern in the problem (see Figure 6c). Furthermore, S3 is in the formalising layer, so it appears that the subject can find the concept of a number pattern abstractly to solve the problem correctly. However, S3 experienced folding back to ensure the answer again.

Figure 6d shows S3 is in the observing layer, where the subject can capture a pattern from the observations made and pay attention to the previous pattern to solve the problem given. To see S3's understanding of solving literacy problems, the results of the completion activity interview are presented.

Researcher	:	*How did you solve the test question?*
S3	:	*I solved the problem (a) by first looking at the picture; the lower the number of boxes, the more* ***(primitive knowing).*** *Starting from that, I see that the last image is the 4th row with 4 squares. Therefore, the next rows are 5 and 6. The 5th row is 5 squares and the 6th row is 6 squares* ***(image making).***
Researcher	:	*For (b), why did you draw 2 rows?*
S3	:	*Because it looks like the picture is getting to the right and the number of squares is getting smaller and smaller* ***(primitive knowing)****. The leftmost (first) box has an image of 4 rows, the second box has an image of 3 rows, then the third box is 2 rows. Where each box is reduced by one row, the answer for the third box is 2 rows* ***(image having).***
Researcher	:	*Problem (c): why did you answer the 3rd row as 6?*
S3	:	*I answered 6 because I had initially counted the pictures from row 1 to row 3* ***(image making).*** *I counted it from row 1 + row 2 + row 3 = 1 + 2 + 3, which is 6, and so on until the 6th row, which obtained the number of squares as many as 28* ***(property noticing).***
Researcher	:	*For the 7th row, there is no example picture. How did you solve it?*
S3	:	*For line 7, I found that there were different jumps from row 2 to row 6. So that 7th row plus jumps by +7* ***(formalising)****. So obtained: square sum of* 6^{th} *row + jump (+7) → 21+7=28*
Researcher	:	*For question (d), why did you answer like that?*
S3	:	*Each row is added according to the question and can be calculated from the picture of problem (a). I calculated it like this:* *1*st *Row →Row 1 = 1* *2*nd *Row→Row 1+Row 2 = 1+2= 3* *3*rd *Row→Row 1+Row 2+Row 3 = 1+2+3= 6* *4*th *Row→Row 1+Row 2+Row 3+Row 4 = 1+2+3+4 = 10* *5*th *Row→Row 1+Row 2+Row 3+Row 4+Row 5 = 1+2+3+4+5 =15* *6*th *Row →Row 1+Row 2+Row 3+Row 4+Row 5+Row 6=1+2+3+4+5+6 =21* *7*th *Row→Row 1+Row 2+Row 3+Row 4+Row 5+Row 6+Row 7* *=1+2+3+4+5+6+7= 28* ***(observing)***

Based on the results of the work of the test questions and interviews with S3, the mathematical understanding layer can be presented in Figure 7.

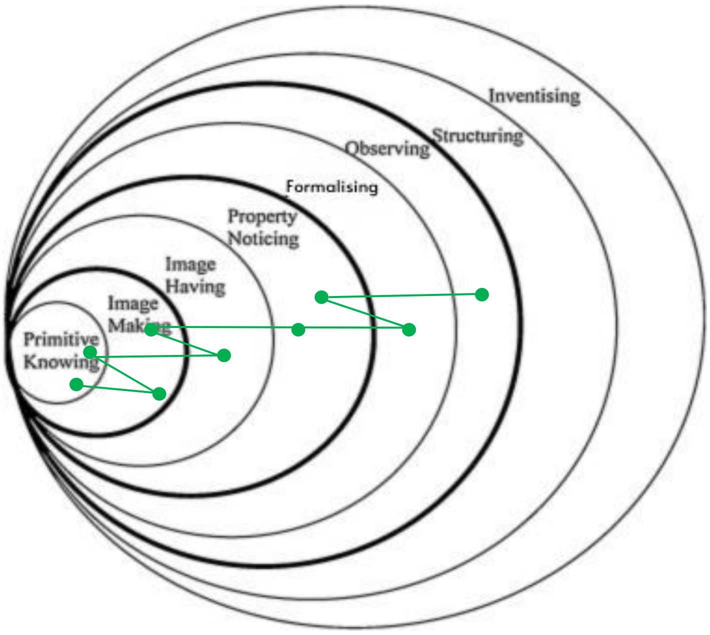


Figure 7. Layers of mathematical understanding S3

3.2. Discussion

Research on students' understanding of solving math literacy problems was analyzed using the Pirie-Kieren theory, which has eight layers with different levels (Pirie & Kieren, 1994). The results show that students' levels of understanding are different, and most activities occur in the image-making and property noticing layers. Figure 8 shows the layers of understanding student mathematics.

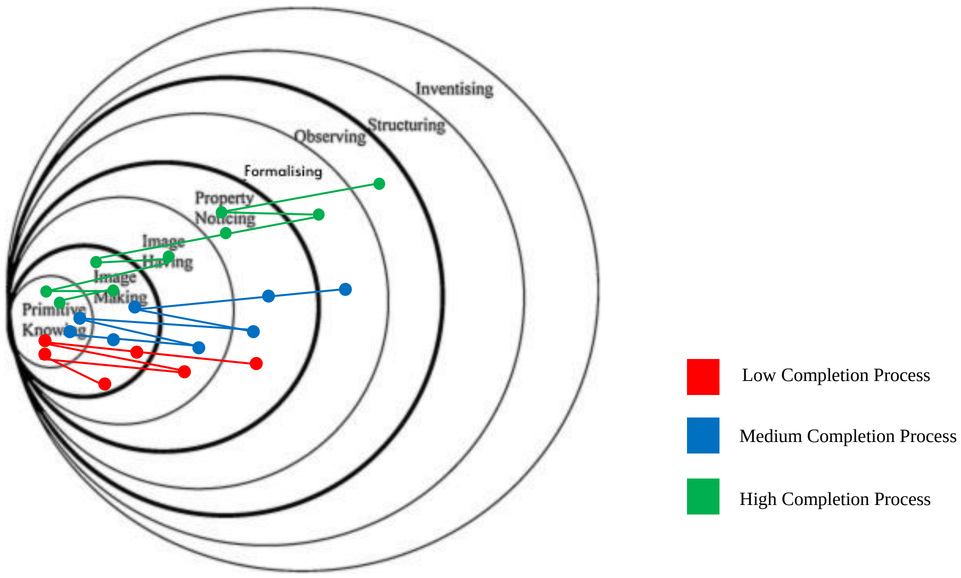


Figure 8. Layers of understanding student mathematics

Based on Pirie-Kieren's theory, at the primitive knowing layer, students can understand the concept of patterns and mention what the question asks. This is linear with Yao and Manouchehri (2022), which explain that students can reach the primitive knowing layer if they understand a concept. In this, students have an initial visual understanding and identify the concept of a number pattern. Students can also identify problems with an increasing or decreasing pattern. This shows that students already have an initial understanding of number patterns. Based on this explanation, students have fulfilled the indicators to reach the primitive knowing layer.

In the image-making layer, students already have a solution idea, so they do not take a long time to solve literacy problems. During the solution process, students can illustrate the fence correctly and connect the pattern items using number symbols to make it easier to answer. Based on this explanation, students fulfill the indicators at the image-making layer, so they have reached this layer. In the image having layers, students already have an image to solve literacy problems coherently. Students can appropriately relate the concept of visually shrinking patterns and continue drawing patterns without engaging in specific activities such as giving number symbols. In line with the opinion of George and Voutsina (2023), to reach the image layer, students have a solution idea and no longer use examples to solve certain problems. Based on this explanation, students fulfill the indicators at the image having a layer, so students have reached this layer.

According to Martin (2008), students can determine characteristics and connect images with a concept in the property noticing layer. Students have been able to determine the characteristics of the pattern in the problem, where the pattern found is an increasing and continuous pattern (+3,+4,+5, +6). This shows that students can determine the characteristics and interrelationships of number patterns. Based on this explanation, students have fulfilled the indicators at the property noticing layer, so students reach this layer.

At the formalising layer, students are said to be able to reach this layer if they acquire new knowledge and find their own concepts to solve problems (Gulkilik, 2016). In this layer, students with medium and high completion processes find new knowledge about the idea of number patterns based on the understanding used to solve the problem. Meanwhile, students with a low completion rate cannot understand the concept of numbers, so the problem cannot be solved. In this case, students with a low completion process cannot identify the previous number pattern to find the number pattern more formally, so the students fail to solve the math literacy problem. In other words, students have not reached the formalising layer level or are still at the noticing layer level. Students have not reached the formalising layer level because they cannot connect mathematical objects and do not pay attention to the properties of prior knowledge to make generalizations about number pattern rules (Güner & Uygun, 2020; Yao & Manouchehri, 2022). Based on this explanation, students with moderate and high completion processes meet the indicators and reach the formalising layer. Only students with a high completion process meet the indicators in the observing layer and reach this layer. The interview results explained that the student noticed the previous pattern, so he could solve the problem. In line with the opinion of Gülkılık et al. (2015), to achieve this layer, students observe the previous pattern and organize their observations. Meanwhile, students with low and medium completion processes didn't do it.

Regarding the structuring and inventising layers, no students achieved this understanding. Students can only reach the sixth layer, the observing. This is because both layers require complex thinking and interpretation skills that students cannot achieve. During the process of solving literacy problems, students often experience folding back. Folding back allows one to expand the lacking mathematical understanding to reorganize the previously built understanding to develop a new understanding according to the topic (Pirie & Kieren, 1994).

Research results from Peñaloza and Vásquez (2022), Ayu et al. (2021), and Pratama (2017) did not use context. This shows that students' understanding only reaches the formalising layer. The context set in the city of Sidoarjo makes students' understanding at the observing stage. Based on this, Sidoarjo-contextualized literacy problems can over-stimulate students' understanding. Therefore, the context of learning should be used in

Sidoarjo. This reinforces the claim of Yaro et al. (2020) as the use of local context in mathematics tasks. In this, local context is a mathematical setting that is authentic, meaningful, and related to local environmental, cultural, and community issues. Yasukawa et al. (2018) mentioned this kind of mathematical setting as local boundaries that students recognize to represent the context that students recognize in learning practices. Hence, future research is recommended to use the local context as a background for literacy problems in learning or problem solving, this is needed to explore and develop layers of understanding to a higher level. Local context for further research means that researchers can conduct further research on authentic tasks that represent environmental or cultural issues that are located and recognized around students.

4. CONCLUSION

The results concluded that the elementary school understanding layer based on Piere-Kieren's theory in solving literacy problems with Sidoarjo context is at the sixth layer, the observing. In solving literacy problems, students have gone through several stages, namely primitive knowing, image making, image having, property noticing, formalising, and observing. Students' level of understanding layer achievement varies, with most understanding layer activities occurring at the level of image-making and property noticing layers. Students with a low completion process can only reach the property noticing layer. Students who have a moderate solution process can reach the formalising layer. Meanwhile, students with a high completion process reach a higher layer than others, the observing layer.

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REFERENCES

Apsari, R. A., Putri, R. I. I., Sariyasa, S., Abels, M., & Prayitno, S. (2020). Geometry representation to develop algebraic thinking: A recommendation for a pattern investigation in pre-algebra class. *Journal on Mathematics Education*, *11*(1), 45-58. https://doi.org/10.22342/jme.11.1.9535.45-58

Ayu, Y. A., Kurniati, Y., Fauziah, A., & Kurnia, R. A. (2021). Analisis kemampuan komunikasi matematis siswa sekolah dasar dalam menyelesaikan soal cerita menerapkan teori Pirie Kieren [Analysis of elementary school students' mathematical communication skills in solving story problems applying Pirie Kieren's theory]. *ISEJ: Indonesian Science Education Journal*, *2*(2), 90-98.

Baumert, J., Nagy, G., & Lehmann, R. (2012). Cumulative advantages and the emergence of social and ethnic inequality: Matthew effects in reading and mathematics development within elementary schools? *Child development*, *83*(4), 1347-1367. https://doi.org/10.1111/j.1467-8624.2012.01779.x

Berisha, V., & Bytyqi, R. (2020). Types of mathematical tasks used in secondary classroom instruction. *International Journal of Evaluation and Research in Education*, *9*(3), 751-758. https://doi.org/10.11591/ijere.v9i3.20617

Brezovszky, B., McMullen, J., Veermans, K., Hannula-Sormunen, M. M., Rodríguez-Aflecht, G., Pongsakdi, N., Laakkonen, E., & Lehtinen, E. (2019). Effects of a mathematics game-based learning environment on primary school students' adaptive number knowledge. *Computers & Education*, *128*, 63-74. https://doi.org/10.1016/j.compedu.2018.09.011

Celik, H. C. (2019). Investigating the visual mathematics literacy self-efficacy (VMLSE) perceptions of eighth grade students and their views on this issue. *International Journal of Educational Methodology*, *5*(1), 165-176. https://doi.org/10.12973/ijem.5.1.177

Chen, M.-J., Lee, C.-Y., & Hsu, W.-C. (2015). Influence of mathematical representation and mathematics self-efficacy on the learning effectiveness of fifth graders in pattern reasoning. *International Journal of Learning, Teaching and Educational Research*, *13*(1), 1-16.

Clair, J. S. (2018). Using cartoons to make connections and enrich mathematics. In *Proceedings of the Interdisciplinary STEM Teaching and Learning Conference* (pp. 86-111). https://doi.org/10.20429/stem.2018.020112

Creswell, J. W. (2012). *Educational Research: Planning, Conducting and evaluating Quantitative and Qualitative Research* (4th ed.). Pearson.

Efriani, A., Putri, R. I. I., & Hapizah, H. (2019). Sailing context in PISA-like mathematics problems. *Journal on Mathematics Education*, *10*(2), 265-276. https://doi.org/10.22342/jme.10.2.5245.265-276

Genc, M., & Erbas, A. K. (2019). Secondary mathematics teachers' conceptions of mathematical literacy. *International Journal of Education in Mathematics, Science and Technology*, *7*(3), 222-237.

George, L., & Voutsina, C. (2023). Children engaging with partitive quotient tasks: Elucidating qualitative heterogeneity within the image having layer of the Pirie–Kieren model. *Mathematics Education Research Journal*. https://doi.org/10.1007/s13394-023-00461-1

Gulkilik, H. (2016). The role of virtual manipulatives in high school students' understanding of geometric transformations. In P. S. Moyer-Packenham (Ed.), *International perspectives on teaching and learning mathematics with virtual manipulatives* (pp. 213-243). Springer International Publishing. https://doi.org/10.1007/978-3-319-32718-1_10

Gülkılık, H., Uğurlu, H. H., & Yürük, N. (2015). Examining students' mathematical understanding of geometric transformations using the pirie-kieren model. *Educational Sciences: Theory & Practice*, *15*(6), 1531-1548.

Güner, P., & Uygun, T. (2020). Examining students' mathematical understanding of patterns by Pirie-Kieren model. *Hacettepe Üniversitesi Eğitim Fakültesi Dergisi*, *35*(3), 644-661. https://doi.org/10.16986/HUJE.2019056035

Holenstein, M., Bruckmaier, G., & Grob, A. (2021). Transfer effects of mathematical literacy: an integrative longitudinal study. *European Journal of Psychology of Education*, *36*(3), 799-825. https://doi.org/10.1007/s10212-020-00491-4

Kholid, M. N., & Nissa, M. (2022). Students' math literacy in solving PISA-like problems in Papuan local context. *AL-ISHLAH: Jurnal Pendidikan*, *14*(4), 5645-5656. https://doi.org/10.35445/alishlah.v14i4.2258

Kolar, V. M., & Hodnik, T. (2021). Mathematical literacy from the perspective of solving contextual problems. *European Journal of Educational Research*, *10*(1), 467-483. https://doi.org/10.12973/eu-jer.10.1.467

Kusuma, D., Sukestiyarno, Y. L., Wardono, W., & Cahyono, A. N. (2022). The characteristics of mathematical literacy based on students' executive function. *European Journal of Educational Research*, *11*(1), 193-206. https://doi.org/10.12973/eu-jer.11.1.193

Manoy, J. T., & Purbaningrum, M. (2021). Mathematical literacy based on ethnomathematics of Batik Sidoarjo. *Jurnal Didaktik Matematika*, *8*(2), 160-174. https://doi.org/10.24815/jdm.v8i2.21644

Martin, L. C. (2008). Folding back and the dynamical growth of mathematical understanding: Elaborating the Pirie–Kieren Theory. *The Journal of Mathematical Behavior*, *27*(1), 64-85. https://doi.org/10.1016/j.jmathb.2008.04.001

NCTM. (2000). *Principles and standards for school mathematics*. NCTM.

OECD. (2019). *PISA 2018 Results (Volume I): What students know and can do*. OECD Publishing.

Peñaloza, J. A., & Vásquez, F. M. R. (2022). Understanding ratio through the Pirie-Kieren model. *Acta Scientiae*, *24*(4), 24-56. https://doi.org/10.17648/acta.scientiae.6826

Pirie, S., & Kieren, T. (1994). Growth in mathematical understanding: How can we characterise it and how can we represent it? *Educational Studies in Mathematics*, *26*(2), 165-190. https://doi.org/10.1007/BF01273662

Posicelskaya, M. A., Rudchenko, T. A., & Semenov, A. L. (2023). Mathematical elements of elementary education. *Doklady Mathematics*, *107*(1), S10-S41. https://doi.org/10.1134/S1064562423700576

Pratama, N. A. E. (2017). Perkembangan pemahaman matematis siswa sekolah dasar kelas V berdasarkan teori Pirie-Kieren pada topik pecahan [Development of fifth grade elementary school students' mathematical understanding based on the Pirie-Kieren theory on the topic of fractions]. *Sekolah Dasar: Kajian Teori dan Praktik Pendidikan*, *26*(1), 77-88. https://doi.org/10.17977/um009v26i12017p077

Rahayuningsih, S., Sa'dijah, C., Sukoriyanto, S., & Qohar, A. (2022). Exploring students' understanding layers in solving arithmagon problems. *Cakrawala Pendidikan: Jurnal Ilmiah Pendidikan*, *41*(1), 170-185. https://doi.org/10.21831/cp.v41i1.33837

Rojas, E., & Benakli, N. (2020). Mathematical literacy and critical thinking. In J. C. But (Ed.), *Teaching college-level disciplinary literacy: Strategies and practices in STEM and professional studies* (pp. 197-226). Springer International Publishing. https://doi.org/10.1007/978-3-030-39804-0_8

Rusdiana, R., Samsuddin, A. F., Muhtadin, A., & Fendiyanto, P. (2023). Development of mathematical literacy problems using East Kalimantan context. *Jurnal Cendekia: Jurnal Pendidikan Matematika*, *7*(1), 197-210. https://doi.org/10.31004/cendekia.v7i1.1885

Stacey, K., & Turner, R. (2015). *Assessing mathematical literacy: The PISA experience*. Springer Cham. https://doi.org/10.1007/978-3-319-10121-7

Suindayati, S., Afifah, D. S. N., & Suja'i, I. S. (2019). Teori Pirie-Kieren: Lapisan pemahaman siswa smp berkemampuan matematika tinggi dalam menyelesaikan soal bangun ruang [Pirie-Kieren theory: Layers of understanding of junior high school students with high mathematical abilities in solving geometric problems]. *MaPan: Jurnal matematika dan Pembelajaran*, *7*(2), 211-228. https://doi.org/10.24252/mapan.2019v7n2a4

Susanta, A., Sumardi, H., Susanto, E., & Retnawati, H. (2023). Mathematics literacy task on number pattern using bengkulu context for junior high school students. *Journal on Mathematics Education*, *14*(1), 85-102. https://doi.org/10.22342/jme.v14i1.pp85-102

Umbara, U., & Suryadi, D. (2019). Re-interpretation of mathematical literacy based on the teacher's perspective. *International Journal of Instruction*, *12*(4), 789-806. https://doi.org/10.29333/iji.2019.12450a

Wagner, D. A. (2011). What happened to literacy? Historical and conceptual perspectives on literacy in UNESCO. *International Journal of Educational Development*, *31*(3), 319-323. https://doi.org/10.1016/j.ijedudev.2010.11.015

Yao, X., & Manouchehri, A. (2022). Folding back in students' construction of mathematical generalizations within a dynamic geometry environment. *Mathematics Education Research Journal*, *34*(2), 241-268. https://doi.org/10.1007/s13394-020-00343-w

Yaro, K., Amoah, E., & Wagner, D. (2020). Situated perspectives on creating mathematics tasks for peace and sustainability. *Canadian Journal of Science, Mathematics and Technology Education*, *20*, 218-229. https://doi.org/10.1007/s42330-020-00083-w

Yasukawa, K., Rogers, A., Jackson, K., & Street, B. V. (2018). *Numeracy as social practice: Global and local perspectives* (1st ed.). Routledge. https://doi.org/10.4324/9781315269474