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FOSTERING MATHEMATICAL CONNECTIONS AND HABITS OF MIND: A PROBLEM-BASED LEARNING MODULE FOR ELEMENTARY EDUCATION

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ABSTRACT

Previous research has highlighted the challenges of improving students' mathematical skills, particularly in connecting mathematical concepts to everyday life and developing strong mathematical habits of mind. This research aims to integrate problem-based learning (PBL) into a mathematics module to enhance mathematical connection abilities and mathematical habits of mind. The ADDIE method was employed to achieve this goal, encompassing five stages: Analysis, Design, Development, Implementation, and Evaluation. The findings indicate that media, material, and language experts deemed the module highly suitable. Furthermore, the product was rated as highly practical based on teacher assessments, student responses, and observations during implementation. Effectiveness tests of the product, conducted through MANOVA, t-tests, and N-gain tests, revealed that the module is highly effective in enhancing students' mathematical connection abilities and moderately effective in improving their mathematical habits of mind. These findings underscore the importance of integrating digital learning technologies to increase the module's engagement and accessibility.

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1. INTRODUCTION

In the literature, mathematical connections have emerged as a compelling topic in various educational mathematics research agendas. The significance of these connections is widely recognized and represents one of the primary goals of mathematics education in the curricula of many countries (Rodríguez-Nieto et al., 2022). The ability to make mathematical connections is crucial for students to achieve a profound understanding of mathematics (García-García & Dolores-Flores, 2018; Muhtarom et al., 2021; Rachmawati et al., 2021).

In other words, students who can interlink various mathematical concepts and apply them in real-world situations tend to have a better grasp of the subject. Integrating mathematics with other subjects allows students to perceive the relevance and application of mathematical concepts in diverse contexts. This integration is pertinent to the aspect of mathematical connections, which includes connections within mathematical components, the relationship between mathematics and other fields of study, and the connection between mathematics and everyday life (NCTM, 2000).

There is a notable lack of empirical data on these mathematical connections among elementary school students in Indonesia. PISA surveys indicate that Indonesian students' ability to make mathematical connections is low (OECD, 2019, 2023; Wijaya et al., 2024). This assertion is supported by test results from our preliminary study sample in four elementary schools in Yogyakarta, which showed an average of 24%, indicating a low level of mathematical connection ability (Nabillah et al., 2023). Cogntive interviews also revealed that only 10% of students can connect mathematical concepts, 12% can relate mathematics to other fields, and 15% can link mathematics to everyday life. The preliminary study highlights the need for targeted interventions and improvements in mathematics teaching and learning to address students' deficiency in mathematical connection ability, which could impact their future academic and professional success, as they may struggle to apply mathematical concepts to real-world problems.

Numerous studies (e.g., Rafi & Retnawati, 2022) indicate that students' mathematical habits of mind may have an impact on their low proficiency in mathematical connections. These habits are essential thinking processes for understanding and engaging in mathematics. Habits in mathematics involve recognizing patterns, experimenting, accurately describing concepts, inventing new methods, visualizing ideas, conjecturing, logical deduction, and abstracting, fostering exploratory and innovative approaches (Cuoco et al., 1996; Gordon, 2011). Addressing these habits through targeted interventions can help students develop the necessary skills to make mathematical connections and apply their knowledge effectively. Gordon (2011) emphasizes the importance of developing mathematical habits of mind, suggesting that these habits are crucial for a deep understanding of mathematics, which aligns with our focus on enhancing mathematical connections. Mathematical habits of mind also encompass affective abilities related to mathematical connection skills, reflecting students' habitual mathematical thinking in various contexts (Andriani et al., 2017). These habits influence students' motivation to learn mathematics and their ability to solve mathematical problems (Kurniansyah et al., 2022; Maarif & Fitriani, 2023; Yustinah et al., 2023). Therefore, the development of mathematical habits of mind is crucial for elementary school students (Bülbül, 2021).

Classroom observations and student interviews have shown that fourth grade in our preliminary study sample of elementary school students currently lack mathematical habits of mind. They exhibit difficulties, tend to complain easily, give up when facing mathematical problems, perceive mathematics as difficult, struggle to describe problem meanings, and resist considering others' opinions in problem-solving. Consequently, the problems identified in this study are twofold: the low ability in mathematical connections and the absence of mathematical habits of mind among students. Both of these issues can be traced back to insufficient instructional resources and support tools. Analysis of our preliminary study results reveals that the Grade 4 mathematics textbook in the independent curriculum, especially for whole numbers, is underutilized. The mathematics textbook in the independent curriculum is not in line with the learning objectives of the curriculum, contains material deemed too difficult for fourth graders, and provides few conceptual explanations for the questions. These problems with the independent curriculum mathematics textbook have led to three out of four schools in our sample not utilizing it. To address these identified needs and challenges, researchers have developed a mathematics learning module focused on whole numbers. Modules are instructional resources and learning aids that can be independently used to achieve educational objectives (Nardo, 2017). Modules play a crucial role in the success of mathematics education.

The development of this module is based on a problem-based learning approach. Problem-based learning is defined as an approach in teaching children's science that can enhance various skills possessed by students (Moallem et al., 2019). In problem-based learning, students examine problems based on their existing knowledge, fostering deep understanding (Moust et al., 2005). Problem-based learning features five key characteristics: student-centered learning, the use of authentic problems, self-directed learning to acquire new information, small group learning, and the teacher's role as a facilitator (Boye & Agyei, 2023; Houghton, 2023; Tan, 2003).

Problem-based learning has been well-documented in the literature as a pedagogical tool capable of enhancing learning outcomes, acquisition of knowledge, higher-order thinking skills (Moallem, 2019), critical thinking skills (Dabbagh, 2019), as well as motivation and interest (Munawaroh et al., 2022; Rotgans & Schmidt, 2019; Wijnia et al., 2024). This positive impact can be observed both at the college level (Bosica et al., 2021; Boye & Agyei, 2023) and at the secondary school level. However, Westwood (2011) argues that although problem-based learning are effective for learning mathematics at the secondary school level, there are many obstacles when applied in primary schools, especially in the early years. Furthermore, he states that there is no strong evidence that most students achieve adequate mastery of basic numeracy skills solely by engaging in problem-based learning. Contrary to Westwood’s opinion, we reference research of Rahmawati et al. (2023) indicating that the more students are exposed to context-based problems, the more proficient they become in practicing their number sense, and vice versa. Number sense and operations enable them to solve numerical problems effectively, efficiently, and flexibly. Therefore, it is crucial for elementary school children to be exposed to authentic problems that highlight the relationship of mathematics with their surrounding world and other science, rather than viewing it as an isolated body of knowledge.

The decision to employ problem-based learning in this mathematics module is based on the alignment of principles between problem-based learning and the focus area under study, namely mathematical connections and mathematical habits of mind. The primary objective of this research is to develop a feasible, practical, and effective problem-based learning-based mathematics module aimed at enhancing the mathematical connection skills and mathematical habits of mind among fourth-grade elementary school students.

2. METHOD

This research uses the ADDIE approach, consisting of five stages: Analysis, Design, Development, Implementation, and Evaluation (see Figure 1). The first stage, preliminary analysis, focuses on identifying problem-causing factors. We have embedded this phase in the background to reinforce what will be needed. The second stage, design, verifies planned methods and designs. This stage is carried out by creating a draft module. The third stage, development, aims to produce a product and validate it through expert assessments and limited student tests. The fourth stage, implementation, measures practicality and effectiveness in the classroom. The fifth stage, evaluation, assesses product quality at all stages of the research. We use this phase not only after the fourth phase is completed, but we carry out formative evaluations at each phase carried out.

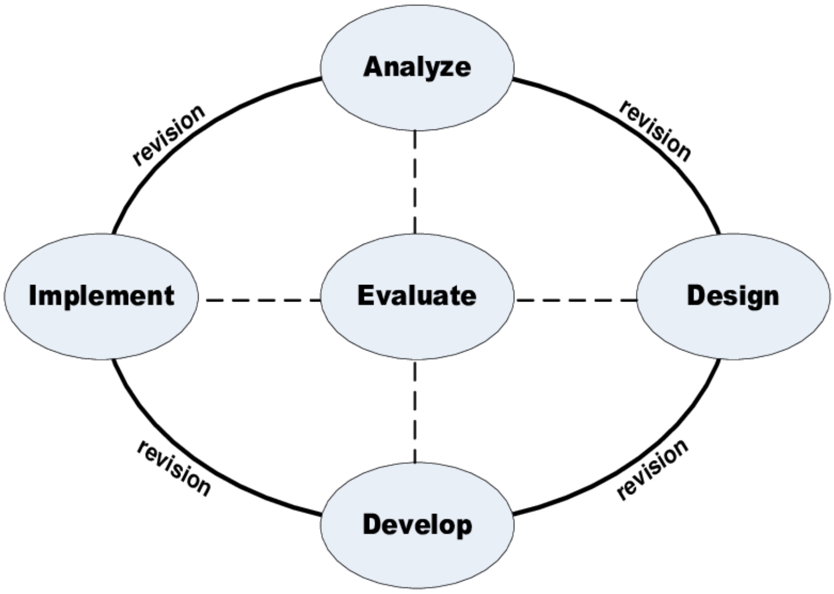


Figure 1. the stage of research development with ADDIE

In third and fourth stages, product trials measure feasibility, practicality, and effectiveness of products, divided into limited and field trials, which are conducted after being declared suitable by experts. The limited trial measures product feasibility by assessing readability and students' understanding of the module, conducted outside class hours. Students provide responses to products and conduct interviews to gather results. Field trials measure product practicality and effectiveness using a non-equivalent pretest-posttest control group design, implementing a product against studied capabilities. Each feasibility, practicality and effectiveness test are explained in more detail as in Table 1.

Table 1. Summary of data, instruments, and forms of analysis

What is Measured	Type	Form of Implementation	Subject	Analysis
Feasibility	Expert Instrument Validation	Feasibility Questionnaire Completion	• Research and educational evaluation experts with doctoral degrees. (Instrument research validation) • Elementary mathematics education experts with doctoral degrees. (Test instrument and ability questionnaire validation)	• Descriptive
	Expert Module Validation	Feasibility Questionnaire Completion (Media, Material, and	• Learning Technology Experts with doctoral degrees	• Descriptive

What is Measured	Type	Form of Implementation	Subject	Analysis
		Language Experts)	• Mathematical Education Experts – Material Experts with doctoral degrees • Language Education Experts with doctoral degrees	
Practicality	Small Scale Trial (One-to-one and small group)	• Product Readability Form Completion • Interviews	• 18 students from a elementary school in Yogyakarta • one elementary school teacher from the same school	• Descriptive
Effectiveness	Implementation	• Quasi-experiment Nonequivalent control group design • Conducted 6 (six) teaching experiments	• Total of 95 fourth-grade students from two public schools in Yogyakarta. • 2 experimental classes (n = 45) and 2 control classes (n = 50)	• MANOVA • Independent sample t-test • N-Gain test

Table 1 indicates that the participants in the practicality study consisted of 18 fourth-grade students from a private elementary school in Yogyakarta and one of their classroom teachers. For the effectiveness study, there were 95 fourth-grade students from two public elementary schools in Yogyakarta, divided into 2 experimental classes (n = 45 students) and 2 control classes (n = 50 students). This effectiveness study adopted a quasi-experiment nonequivalent control group design.

There were 6 teaching experiments for each group, with the experimental classes using the PBL integrated module, while the control classes received conventional learning treatments. These six teaching sessions included: (1) reading, writing, and determining the place value of numbers; (2) comparing and ordering; (3) arithmetic operations of addition; (4) arithmetic operations of subtraction; (5) arithmetic operations of multiplication; (6) arithmetic operations of division. Besides covering these topics, a brief description of the developed PBL integrated module includes several characteristics, such as always starting with a phenomenon part, which is a context of problems familiar to children and includes authentic phenomena around them; followed by two parts, "let's do" and "let's be active." The “let's do” activity involves solving the given problems using the learned concepts, while the “let's be active” activity involves individual or group tasks to support understanding of the learned concepts. Other parts include concept explanations, "let's practice," and "let's play." The “let's practice” activity is an individual task aimed as an evaluation of understanding of the material or discussed concepts, whereas the “let's play” activity contains interesting and educational games for students. Details of the order and variety of module activities can be seen at the link: https://s.id/Modul_BilanganSD.

The feasibility and practicality studies of the module were analyzed using descriptive statistics, which were used to determine the feasibility and practicality criteria of the module.

Data analysis for the effectiveness study used the MANOVA test, independent t-test, and N-gain with prerequisite tests that had been fulfilled previously.

3. RESULT AND DISCUSSION

3.1. Results

This section is composed of four parts: preliminary analysis, feasibility, practicality, and effectiveness results. After each of these research results is presented, it was followed by discussion.

3.1.1. Preliminary analysis

In the preliminary analysis, the study found that students' mathematical connection abilities were low (24%), and they were not accustomed to using mathematical habits of mind. They were used to learning styles exemplified by teachers. Based on this preliminary analysis, a mathematics module based on problem-based learning with grade 4 whole number material for the independent curriculum is proposed. The mathematics module was developed based on module development principles and rooted in problem-based learning. It contains various student adventures and activities as well as diverse concept explanations. This module explains the material of whole numbers and all their arithmetic operations.

3.1.2. Module feasibility

The study evaluates the feasibility of the problem-based learning mathematics module products, considering media, material, and language opinions from experts, with the results presented in Table 2.

Table 2. Expert validation results

No.	Validation	Average (%)	Rating
1	Media Field Expert	92.80	Highly Feasible
2	Material Field Expert	96.00	Highly Feasible
3	Language Field Expert	86.66	Highly Feasible

The mathematics module product, as assessed by experts, is considered very feasible according to the data presented in Table 2. The feasibility assessment of the module from the media aspect is declared highly feasible because it meets several criteria including: appropriate color selection for primary school students, font choice (Arial 12 pt) suitable for readability at the fourth-grade level, and the use of attractive images that explain the text. The feasibility from the material aspect is also rated highly, referring to criteria such as complete material content, alignment with the curriculum, and material accuracy. The language aspect was also deemed highly feasible by experts due to the use of clear, communicative, dialogic language, suitable for student development, and adherence to language rules. Figure 2 is an example of the language found in the phenomenon section of the module, which contains a problem context that can be visualized by students.

Translation in English

PHENOMENON 2

Caca sells clothes at a bazaar with her family. Caca sells clothes at the bazaar for IDR10,000.00 each. If Caca successfully sells 53 pieces of clothing, how much total money can Caca collect? Help Caca count the total money that can be collected!

Figure 2. An example of a problem context in the module

Figure 2 shows an example of a problem context from the phenomenon section of the module used to start learning. The context of the sentence is also adjusted to the student's language and dialect.

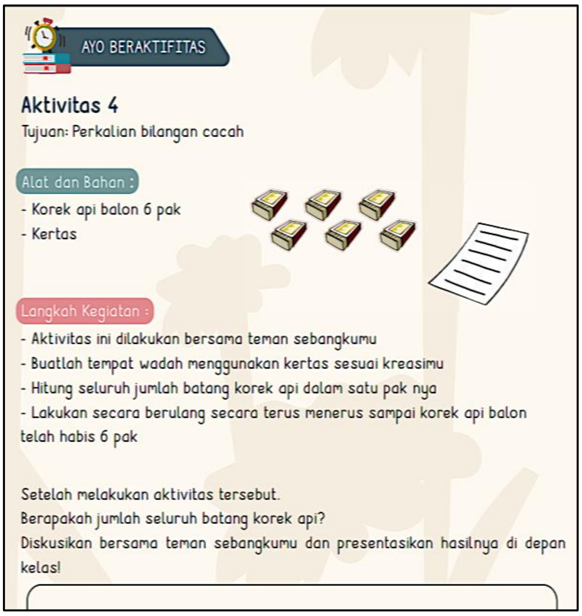
3.1.3. Module practicality

The module's practicality is assessed through teacher evaluation, student responses, and product implementation observations, as shown in Table 3.

Table 3. Product practicality results

No.	Aspect	$\bar{X}$	Rating
1	Teacher Evaluation	87.96	Highly Practical
2	Student Response	81.94	Highly Practical
3	Observation	90.93	Highly Practical

Table 3 indicates that the practicality assessment of the mathematics module product is highly feasible, making it an ideal choice for learning grade 4 mathematics related to whole numbers. The fulfillment of all elements and aspects of practicality concludes that technically this module can be used in teaching. The module includes various direct student activities, with problems derived from real life. Problem-solving is done using various methods and includes individual and group student activities. Figure 3 is one example of an activity that requires students to collaborate in problem-solving and present their results.



Translation in English

Let's be active!
Activity 4
Objective: Multiplication of whole numbers
Tools and Materials:

- Matchbox 6 pack
- Paper

Steps:

- This activity is done with your classmate
- Make a container using paper according to your creativity
- Count the total number of matchsticks in one pack
- Continue counting sequentially through the matchsticks until all 6 packs are finished

After completing these activities,
What is the total number of all matchsticks?
Discuss with your classmate and present the results in front of the class!

Figure 3. An example of a student activity in the module

Figure 3 is a page from the let's be active section. The section activity involves students working together to count matchsticks from six packs and record their findings. They are also encouraged to use paper creatively to make a container for the matchsticks. After completing the activity, students are asked to discuss their findings with their peers and present their results in front of the class. The module is rated as highly practical and ideal for teaching whole numbers. It includes various direct student activities, with problems derived from real life. Problem-solving is done using various methods and includes both individual and group student activities. This aligns with the collaborative nature of the activity shown in the Figure 3.

3.1.4. Module effectiveness

Analysis of the effectiveness of using the mathematics module on mathematical connection abilities and mathematical habits of mind was based on test results (pretest and posttest) and a mathematical habits of mind questionnaire. Descriptive data from the test and questionnaire analysis are summarized in Table 4.

Table 4. Descriptive results of product effectiveness

	Connection ($\bar{X}$)		**MHM ($\bar{X}$)**	
	Control	**Experimental**	**Control**	**Experimental**
Pretest	27.18	26.26	66.61	66.31
Posttest	86.26	38.70	87.02	70.08

Based on Table 4, it is evident that the mathematical connection abilities and mathematical habits of mind improved in the experimental class as a result of implementing the developed modules. In terms of mathematical connection abilities, the experimental class showed an average score of 27.18 in the pre-test and 86.26 in the post-test, whereas the control class had an average score of 26.26 in the pre-test and 38.70 in the post-test. Regarding mathematical habits of mind, the experimental class had average scores of 66.61

in the pre-test and 87.02 in the post-test, while the control class had average scores of 66.31 in the pre-test and 70.08 in the post-test.

Furthermore, the effectiveness of the mathematics module was assessed through statistical tests using the SPSS application, which had previously undergone prerequisite tests, as summarized in Table 5.

Table 5. Test analysis results for hypothesis

No.	Statistical Test	Results
1	MANOVA	Sig. 000
2	Independent sample t-test (Mathematical Connection)	Sig. 000
3	Independent sample t-test (MHM)	Sig. 000

Based on Table 5, it can be concluded that the significant values in the three tests are below 0.05, indicating a significant difference between the experimental class and the control class. Therefore, it can be concluded that this mathematics module can enhance mathematical connection abilities and mathematical habits of mind. The N-gain test was used to measure the improvement in connection abilities and mathematical habits, with the results presented in Table 6.

Table 6. N-Gain test results

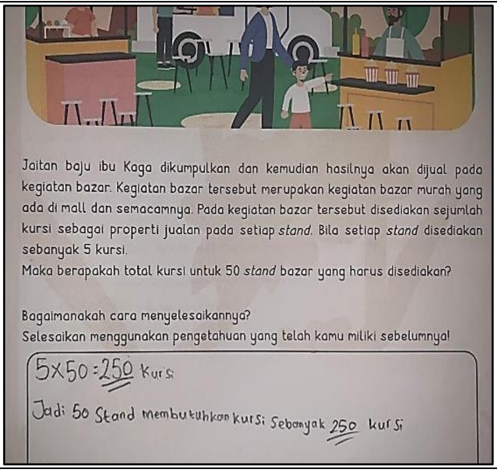
	Connection		MHM	
	Control	**Experimental**	**Control**	**Experimental**
$\bar{X}$	16.71	81.06	64.23	69.90
Classification	Ineffective	Effective	Moderately Effective	Moderately Effective

As can be seen from Table 6, the problem-based learning-based mathematics module is reasonably effective at strengthening mathematical habits of mind and effective at fostering mathematical connection abilities.

3.2. Discussion

The research findings have demonstrated that the role of PBL integrated into modules can enhance mathematical connection and mathematical habits of mind. PBL focuses on problem-solving and applying concepts in real contexts, allowing students to link various mathematical concepts and apply them in practical situations (McFeetors & Palfy, 2018). This reinforces a deep understanding of mathematics as an interconnected web of concepts, not just isolated facts or procedures. Smith and Ragan (2004) state that the importance of context in learning, showing that students are more likely to understand and remember concepts when they can relate the material to real-world situations. PBL, with its case- and problem-oriented design, facilitates these connections, improving cognitive skills and the ability to apply mathematical knowledge in various contexts.

Through this module, which is facilitated in the content within the module, students can understand the material in its implementation with everyday life. This means that students are trained to develop their mathematical connection abilities. An example of student work in the problem-based learning mathematics module can be seen in the Figure 4.


Jaitan baju ibu Kaga dikumpulkan dan kemudian hasilnya akan dijual pada kegiatan bazar. Kegiatan bazar tersebut merupakan kegiatan bazar murah yang ada di mall dan semacamnya. Pada kegiatan bazar tersebut disediakan sejumlah kursi sebagai properti jualan pada setiap *stand*. Bila setiap *stand* disediakan sebanyak 5 kursi.
Maka berapakah total kursi untuk 50 *stand* bazar yang harus disediakan?

Bagaimanakah cara menyelesaikannya?
Selesaikan menggunakan pengetahuan yang telah kamu miliki sebelumnya!

5x50=250 kursi
Jadi 50 Stand membutuhkan kursi sebanyak 250 kursi

Translation in English:
The cloth rags that Kaga collected will later be sold at a bazaar. The bazaar event is a charity bazaar either in the mall or elsewhere. In the bazaar activity, a number of chairs are provided as property at each stand. If each stand is provided with 5 chairs. Then how many total chairs are needed for 50 bazaar stands?

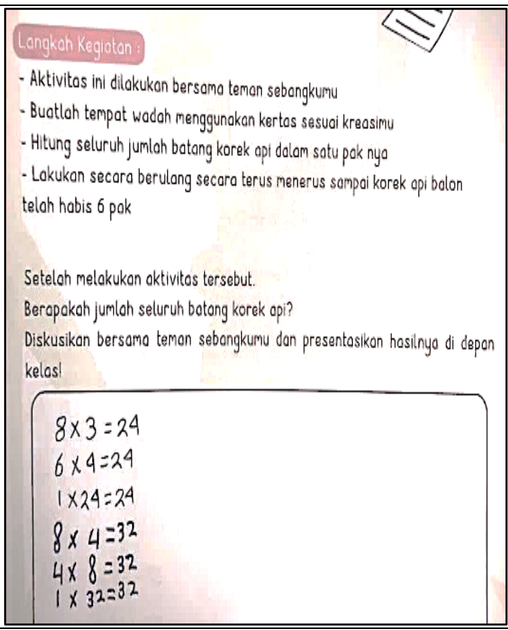
How do you solve it?
Complete using the knowledge you have previously!

5 × 50 = 250 chairs
So, 50 Stands require: As many as 250 chairs.

Figure 4. A sample of student work

The student work shown in the Figure 4 indicates that students are given a mathematical phenomenon from everyday life, and then they solve the problem/phenomenon into a mathematical form in the column. Understanding the implementation of mathematical concepts in everyday life is important because it helps students see the practical relevance of what they are learning. By connecting mathematical concepts to real-world situations, students can develop a deeper understanding of the subject and appreciate its applicability (Ayuningtyas et al., 2024; Harisman et al., 2023; Kusmaryono et al., 2024; Polman et al., 2021; Purnomo et al., 2019). This can also help students develop critical thinking and problem-solving skills, as they learn to apply mathematical principles to solve real-life problems. Overall, developing mathematical connection abilities can help students become more proficient in mathematics and better equipped to apply their knowledge in various contexts.

The activities in the developed module also facilitate students to develop mathematical connection abilities and mathematical habits of mind. An example of an activity and a student's answer can be seen in Figure 5.


Langkah Kegiatan :
- Aktivitas ini dilakukan bersama teman sebangkumu
- Buatlah tempat wadah menggunakan kertas sesuai kreasimu
- Hitung seluruh jumlah batang korek api dalam satu pak nya
- Lakukan secara berulang secara terus menerus sampai korek api balon telah habis 6 pak

Setelah melakukan aktivitas tersebut.
Berapakah jumlah seluruh batang korek api?
Diskusikan bersama teman sebangkumu dan presentasikan hasilnya di depan kelas!

8x3=24
6x4=24
1x24=24
8x4=32
4x8=32
1x32=32

Translation in English:
Activity Steps:
- Do this activity with a friend.
- Create a container using cardboard according to your creativity.
- Count the total number of matchsticks in one box.
- Repeat the activity continuously until six boxes of matchsticks are used up.

After completing the activity: Calculate the total number of all matchsticks from the six boxes.
Discuss with a friend and present the results in front of the class!
8 × 3 = 24
6 × 4 = 24
1 × 24 = 24
8 × 4 = 32
4 × 8 = 32
1 × 32 = 32

Figure 5. A sample of student work

In Figure 5, the module work by student one shows that the student is engaging in an activity in the module, following the steps of the activity and solving problems related to multiplication of whole numbers. After understanding the given problem, the student writes down the arithmetic operations they find in the column based on all the activities they have done. This is one of the steps or activities in the problem-based learning mathematics module, namely "Let's be active," and shows that learning is centered on the student.

Group discussions, as illustrated in Figure 6, show how students collaborate to solve given mathematical problems. These activities not only strengthen the understanding of mathematical concepts but also develop communication and collaboration skills.

Figure 6. Group discussion on problems in the PBL-based mathematics module

Figure 6 shows that group discussions occur to solve a problem using the distributed mathematical learning module. These discussion activities embody one of the principles of PBL that learning can be done in groups. This activity not only strengthens the understanding of mathematical concepts but also develops communication and collaboration skills (Li et al., 2022).

Another activity in the module is to summarize concepts and communicate them, as illustrated by Figure 7.

Figure 7. Self-presentation on PBL-based mathematics module

Figure 7 shows activities in communicating ideas facilitated by the problem-based learning mathematics module. As seen in Figure 7, activities in learning designed in the module make students enthusiastic about learning mathematics, interested in engaging with mathematics, and this is evidenced by the impact on improving students' mathematical connection abilities and mathematical habits of mind. The student presentation activities are self-assessment and peer-assessment strategies that are useful for communicating ideas and clarifying understanding orally, as well as self-reflection (Lianghuo & Mei, 2007; Ma et al., 2008; Purnomo, 2015, 2016).

In general, the result of this study support previous study, such as Cotič and Zuljan (2009). Their experimental study involved 179 students, with an average age of nine years, and aimed to investigate the impact of problem-based teaching on problem-solving skills and students' attitudes towards mathematics. Their results indicate that problem-based teaching, which adheres to the principles of problem-based learning (PBL) demonstrates better knowledge, especially in solving more challenging mathematical problems. Another result is that PBL does not decrease students' motivation to learn mathematics even though the explored problems have higher difficulty levels. Other studies on problem-based learning, like Cognitively Guided Instruction by Fennema (1992) and Conceptually Based Instruction by Blöte et al. (2001); Hiebert and Wearne (1992), support our findings, which are that students learn more, understand it better, and are better at solving problems.

4. CONCLUSION

This study reveals that the problem-based learning mathematics module has demonstrated significant potential as an effective educational resource for grade 4 mathematics. Expert evaluations and empirical studies affirm its high feasibility and practicality. The module's success in bolstering mathematical connection abilities and in fostering mathematical habits of mind is evident. To build upon these promising results, future research should prioritize: (1) In-depth investigation into the development of mathematical habits of mind, with a focus on systematic observation and analytical response evaluation during the instructional process; (2) Extension of teaching experiment durations to fully assess and enhance the module's effectiveness, providing students with sufficient time to deeply engage with and master the mathematical concepts presented.

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