

Implementation of SOLO taxonomy and Newman error analysis in first-order differential equation

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Abstract

First-order Ordinary Differential Equation (ODE) has many applications in physics, engineering, biology, economics, and ecology. Therefore, mastering the concepts and methods of solving ODE is essential for students to be able to apply mathematics in solving real-world problems. However, the teaching of first-order ODE has not paid attention to practical applications, so that students have difficulty linking theory with real cases. This study aims to analyze the implementation of the SOLO taxonomy and Newman Error Analysis (NEA) in first-order ODE. The methodology used is a case study. The research subjects consisted of nine students of the mathematics department of FMIPA Universitas Negeri Padang. Data were collected through tests, interviews, and documentation. Then the data were analysed quantitatively and qualitatively. The results showed that there were five errors in solving first-order ODE made by students, namely Reading Errors (RE), Comprehension Errors (CE), Transformation Errors (TE), Process Skill Errors (PE), and Encoding Errors (EE). Some of the causes of these errors include students' low ability to read mathematical symbols, students' inaccuracy, not being able to use algorithms correctly, not mastering the concepts of algebra, differential, and integral, as well as not understanding in determining the systematic solution of the problem and not being accustomed to writing the final answer. This information can be used as a guideline for lecturers in designing strategies and lecture designs for first-order ODE.

Keywords:

Newman error analysis, Ordinary differential equation, SOLO taxonomy

How to Cite:

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1. INTRODUCTION

Ordinary Differential Equations (ODE) is a branch of mathematics that plays an important role in various disciplines. Engineering, science, and economics students need to understand ODE because of their ability to model phenomena that involve change. In physics, ODE is used to model particle motion, the laws of thermodynamics, and fluid dynamics. ODE forms the basis of structural analysis, control system design, and machine

behaviour prediction (Boyce et al., 2017). In biology, ODE is used to study population dynamics, disease spread, and ecology. For example, exponential and logistic growth models are important for understanding human or animal populations. Johnson et al. (2022) study revealed that ODE is used to understand specific ecosystem mechanisms when studying population growth and analysing the decay of radioactive materials. In economics, ODE analyzes the dynamics of investment and economic growth (Blanchard & Fischer, 1989).

Mastering ODE trains students' analytical and logical thinking skills. In solving ODEs, students are invited to understand concepts, develop mathematical models, and analyse solutions that are relevant to the context of the problem. This process enhances problem-solving skills, which are essential in the working world. Understanding the analytical and numerical solutions of ODE allows students to work with software used in various industries, such as MATLAB or Python (Edwards & Penney, 2000). Learning ODE also has aesthetic value in understanding the beauty of mathematics as a universal language. The pattern of water waves can be explained through ODE which shows the harmony between theory and reality.

However, most students struggle to understand the concept of ODE and apply them appropriately to solve problems (Boyce et al., 2017). One of the main difficulties is the lack of conceptual understanding of the basic concepts of ODE, such as the meaning of derivatives, general and special solutions, and the method of separating variables. Some students simply memorise the solution steps without understanding the meaning behind them, so they struggle when faced with non-routine problems. For example, they often misinterpret initial conditions or parameters in equations as just numbers without understanding them as part of the physical or mathematical context of the problem (Edwards & Penney, 2000).

Another difficulty is algebraic manipulation skills. ODE requires skills in integration, differentiation, and manipulation of complex mathematical expressions. Students who don't master these fundamentals tend to make mistakes in the calculation process, such as incorrectly performing substitutions or misunderstanding integral rules. These errors can lead to irrelevant or even nonsensical solutions (Blanchard et al., 2012). ODE is used to model dynamic phenomena, such as temperature changes, oscillatory motion, or population growth. However, turning real-world problems into mathematical models requires deep analytical skills and multidisciplinary understanding. Students have difficulty identifying variables, parameters, and relationships in a phenomenon, making them unable to construct appropriate equations (Zill, 2009).

The SOLO taxonomy is designed to evaluate the quality of students' learning outcomes based on the complexity of their understanding (Korkmaz & Unsal, 2017). Developed by John Biggs and Kevin Collis in 1982, this taxonomy categorises learning abilities into five hierarchical levels that reflect the progression of understanding from simple to complex (Biggs et al., 2022). At the pre-structural level, students don't yet have a relevant understanding of the material. The answers given are usually not related to the context of the problem or contain fundamental errors. This level indicates that students don't understand what to do (Korkmaz & Unsal, 2017; Potter & Kustra, 2012). At the uni-structural level, students understand one important aspect of the material but it is still limited.

They can identify or use one element in a given task, but cannot integrate it with other elements (Biggs et al., 2022).

Students at the multi-structural level understand some aspects of the material separately (Biggs et al., 2022). However, they are not yet able to connect these elements into a complete picture. Learning outcomes at this stage tend to be an accumulation of facts without deep connections. At the relational level, students connect different aspects of the material in a logical and comprehensive manner. They are able to understand the relationship between the elements and use this understanding to explain or solve problems more effectively (Biggs et al., 2022). The extended abstract level shows students' ability to generalise and transfer learned concepts into new contexts. They can utilise their understanding to think abstractly, create new ideas, or develop theories (Biggs et al., 2022; Brabrand & Dahl, 2009).

The SOLO taxonomy provides guidance for educators to design lessons that focus on the development of students' understanding (Biggs et al., 2022). Using this framework, educators can assess the quality of students' answers based on their level of understanding, design questions or tasks that encourage students to move from uni-structural to extended abstract levels, and help students build critical and analytical thinking skills through a tiered learning approach (Özdemir & Yıldız, 2016).

In addition, Newman's errors are used to analyze students' errors in solving mathematical problems. This framework is a useful tool for educators to diagnose and understand where students make errors so that they can design more effective learning interventions (Newman, 1977). Reading Errors (RE) occur when students are unable to read or understand the text in a maths problem. This problem is often experienced by students with limited literacy so that they fail to identify important information from the problem. For example, students misread the numbers or mathematical symbols presented. In Comprehension Error (CE), students are able to read the problem but fail to understand what is being asked. Students do not understand the context or purpose of the problem so they cannot determine the right first step to solve it.

Transformation Errors (TE) occur when students are unable to transform the information in the problem into an appropriate mathematical model. For example, students incorrectly determine the formula to be used or incorrectly identify relevant variables. In Process Skill Errors (PE), students understand the problem and know the method to use but make errors in calculations or algebraic manipulations. These errors include errors in basic mathematical operations or the use of algorithms. Finally, Encoding Errors (EE) occur when students have found the correct answer but don't write it according to the required format. Students don't include units or incorrectly rearrange the final result.

NEA has been widely used in mathematics education research to diagnose student difficulties and identify areas that require intervention (Mubarakah & Amir, 2024; Wardhani & Argaswari, 2022; Yarman et al., 2024). This analysis is often applied to understand students' difficulties in problem solving geometry, algebra, trigonometry and differential equations. By understanding the types of errors that occur, teachers can adjust their teaching methods accordingly. For example, strengthening mathematical literacy to address RE or using problem-based learning strategies to reduce TE.

The relationship between SOLO taxonomy and NEA lies in analyzing the progression of student understanding and the types of errors that occur at each level. Students at pre-structural and uni-structural levels tend to make RE and CE. Meanwhile, students at the multi-structural and relational levels often encounter TE and PE as they start working with more complex models and methods. By integrating these two frameworks, educators can diagnose students' abilities more deeply and design focused learning strategies. For students at the early level (pre-structural or uni-structural), learning is focused on strengthening mathematical literacy and understanding basic concepts. On the other hand, for students at higher levels (relational or extended abstract), learning is directed towards solving contextual problems and applying more abstract mathematical models.

Therefore, it is important to conduct research that examines the implementation of SOLO taxonomy and NEA in first-order ODE. SOLO taxonomy and Newman errors are complementary approaches in analyzing ODE learning. By understanding the level of student understanding and the types of errors that often occur, educators can create adaptive and effective learning strategies that help students overcome difficulties and achieve deeper understanding. The research questions raised are what are the students' answers in solving ODE and what are the factors that cause them?

2. METHOD

2.1. Research Design

A case study was used in this research. This method is used to gain an in-depth understanding of a complex and unique case. Case studies pay special attention to the dynamics, complexity, and specific details of a case that usually involves a particular individual, group, organization, or event (Yin, 2017).

2.2. Subject

This study began by involving 30 students of the mathematics department of FMIPA Universitas Negeri Padang who took the ODE course. Of the 30 students, nine students were selected through purposive sampling based on their test answers with details of three from high, medium, and low ability. Grouping subjects based on test results, namely high ($\text{Value} \geq \text{Mean} + \text{SD}$), medium ($\text{Mean} - \text{SD} \leq \text{Value} < \text{Mean} + \text{SD}$), and low ($\text{Value} < \text{Mean} - \text{SD}$) groups (Yarman et al., 2024).

2.3. Instrument

The instruments used were tests and interview guidelines. The test given was in the form of two essay questions. In the first problem, a homogeneous differential equation is given and then students determine the solution starting with investigating whether the equation is separable differential equations. In the second problem, students are asked to determine the solution of a non exact differential equation, by first identifying whether the given equation is an exact differential equation or not. The validity of the two problems used was carried out by discussing them with the team of lecturers teaching the ODE course.

Interviews were conducted to gather further information about the errors made by students and the factors that caused them.

2.4. Research Procedure

The students involved in this study totalled 30 students and had attended ODE courses. The material they learnt included first-order ODE. To evaluate the lecture process, the following test was given.

- a. Find the solution of the differential equation $y\sqrt{x^2 + y^2}dx - x(x + \sqrt{x^2 + y^2})dy = 0$ by following these steps.
 - (1) Investigate whether these differential equations are separable differential equations.
 - (2) If it is true that it is separable differential equations, determine the general solution.
 - (3) If the differential equation is non-separable differential equations, prove that it is a homogeneous differential equation.
 - (4) If the differential equation is homogeneous, then transform this equation into separable differential equations.
 - (5) After you have obtained the separable differential equations in part (4), please proceed to find the general solution of the differential equations.

- b. Find the solution of the differential equation $(3x^2 + y^2)dx - 2xydy = 0$ by following these steps.
 - (1) Is this differential equation an exact differential equation?
 - (2) If it is an exact differential equation, find its general solution.
 - (3) Determine the integration factor if it results in a non-exact differential equation.
 - (4) Use the integration factor obtained in part (2) to show that the resulting differential equation is exact.
 - (5) Find the general solution of the exact differential equation in part (3).

Furthermore, students' answers were scored and categorized based on SOLO taxonomy and NEA. From the results of the analysis, nine students representing each of the high, medium, and low abilities were selected to be interviewed. The purpose was to confirm the answers written by the students and to explore the causes of the errors they made.

2.5. Analysis

Test data was analyzed quantitatively to evaluate students' understanding of first-order ODE. Descriptive techniques were used to describe the interview data. There are three stages in analyzing qualitative data, namely reducing data, presenting data, and drawing conclusions. Reducing data is a process of selecting, focusing and transforming raw data obtained through interviews. Presentation of data is done in the form of a brief description and then continued with drawing conclusions (Miles et al., 2014).

3. RESULTS AND DISCUSSION

3.1. Results

The results of data analysis based on the test results of each research subject in the first-order ODE course based on the high, medium, and low academic ability categories according to NEA are summarized in [Table 1](#).

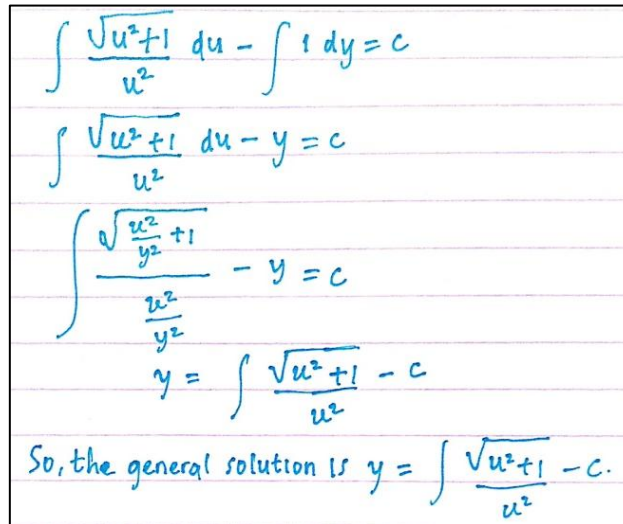
Table 1. Students' NEA in first-order ODE

Group	Subject Codes	Question Number 1					Question Number 2				
		RE	CE	TE	PE	EE	RE	CE	TE	PE	EE
High	H ₁	-	-	-	√	√	-	-	-	-	√
	H ₂	-	-	-	√	√	-	-	-	√	√
	H ₃	-	-	√	√	√	-	√	-	√	√
Medium	M ₁	-	√	√	√	√	-	-	√	√	√
	M ₂	-	√	√	√	√	-	√	√	√	√
	M ₃	-	-	√	√	√	-	√	√	√	√
Low	L ₁	√	√	√	√	√	-	√	√	√	√
	L ₂	-	√	√	√	√	√	√	√	√	√
	L ₃	√	√	√	√	√	√	√	√	√	√

Based on [Table 1](#), the following describes the mistakes made by students in working on questions number 1 and 2.

High Ability Subjects (H₁)

In questions number 1a, 1b, and 1c, subject H₁ can solve all the questions well. In terms of SOLO taxonomy, it means that H₁ didn't make mistakes at the pre-structural, uni-structural, and multi-structural levels. According to NEA, H₁ didn't make RE, CE, and TE. However, H₁ made mistakes on questions number 1d and 1e, namely questions with relational level and extended abstract level. In this case, H₁ made errors in PE and EE. To confirm the errors and find out the factors that caused the mistakes made by subject H₁ in questions 1d and 1e, a snippet of the answers to the questions made by H₁ is shown in [Figure 1](#) followed by an interview.



$$\int \frac{\sqrt{u^2+1}}{u^2} du - \int 1 dy = c$$

$$\int \frac{\sqrt{u^2+1}}{u^2} du - y = c$$

$$\int \frac{\sqrt{\frac{u^2}{y^2}+1}}{\frac{u^2}{y^2}} - y = c$$

$$y = \int \frac{\sqrt{u^2+1}}{u^2} - c$$

So, the general solution is $y = \int \frac{\sqrt{u^2+1}}{u^2} - c$.

Figure 1. H₁'s answers for questions 1d and 1e

In Figure 1, it can be seen that H₁ had problems in calculating the integral in solving problems 1d and 1e. The problem experienced by H₁ is in accordance with the indicators of NEA, which is wrong in operating calculations in solving problems. In this case, the error experienced by H₁ is categorized as PE. In addition, H₁'s final answer was not correct due to errors in the previous process. This is in accordance with NEA indicators, namely incorrectly determining the final answer or not being able to determine the final answer. In H₁'s final answer, the error is categorized as EE. To find out more about PE and EE by H₁ in the answers to questions number 1d and 1e, the following are excerpts of the researcher's interview (R) with H₁.

R : In your answer to question 1, you can see that questions 1a, 1b, and 1c can be answered correctly, why did you make a mistake for question 1d?

H₁ : I found the answer to 1d requires an integral calculation.

R : What were your difficulties in calculating the integral?

H₁ : I struggled to choose the right method for the integral of the function.

R : The answer you wrote is still in the integral sign, have you tried to find the answer, for example by substitution or making algebraic modifications?

H₁ : I've tried several methods but no results, sir.

R : How about your answer to question 1e?

H₁ : Because the answer to question 1e is related to the answer to question 1d so the answer to 1e is also not obtained, only written in integral form, sir.

From the test results and interviews with H₁ described, it is true that H₁ made PE dan EE. The cause of these errors is that H₁ cannot use the algorithm correctly, doesn't master the concepts of algebra, differential, and integral, and doesn't understand in determining the systematic solution of the problem and is not accustomed to writing the final answer.

Next, we will analyze H₁'s mistakes in problem number 2. In questions number 2a, 2b, 2c, and 2d, subject H₁ can solve all the questions well. In terms of SOLO taxonomy, it means that H₁ didn't make mistakes at the pre-structural, unistructural, multi-structural, and

relational levels. According to NEA, H₁ didn't make RE, CE, TE, and PE. However, H₁ made an error on question number 2e with the extended abstract level, namely EE.

$$\left(3 + \frac{y^2}{x^2}\right)dx - \frac{2y}{x}dy = 0$$

$$M_1(x,y) = 3 + \frac{y^2}{x^2} \rightarrow \frac{\partial M_1(x,y)}{\partial y} = \frac{2y}{x^2}$$

$$N_1(x,y) = -\frac{2y}{x} \rightarrow \frac{\partial M_1(x,y)}{\partial x} = \frac{2y}{x^2}$$

Obtained $\frac{\partial M_1(x,y)}{\partial y} = \frac{\partial N_1(x,y)}{\partial x}$, this differential equation is exact.

→ Finding solutions

$$\frac{\partial F(x,y)}{\partial x} = M_1(x,y) \rightarrow \int 3 + \frac{y^2}{x^2} dx$$

$$F(x,y) = 3x - \frac{y^2}{x} + c(y)$$

$$\frac{\partial F(x,y)}{\partial y} = N_1(x,y)$$

$$\frac{\partial}{\partial y} \left(3x - \frac{y^2}{x} + c(y) \right) = -\frac{2y}{x}$$

$$-\frac{2y}{x} + c'(y) = -\frac{2y}{x} \Rightarrow c'(y) = 0$$

$$c(y) = c_1$$

Obtained:

$$F(x,y) = c$$

$$3x - \frac{y^2}{x} + c(y) = c$$

$$3x - \frac{y^2}{x} = c$$

Thus, the general solution of $(3x^2 + y^2)dx - 2xy dy = 0$ is

$$3x - \frac{y^2}{x} = c.$$

Figure 2. H₁'s answers for questions 2e

Figure 2 showed that H₁ had a problem in determining the solution of the exact ODE. The error occurred in using the relationship between the solution function $F(x,y)$ and the ODE coefficients $M_1(x,y)$ and $N_1(x,y)$, so that the integral calculation performed didn't produce a general solution of the ODE. The problem experienced by H₁ is in accordance with the indicators of NEA, namely incorrectly determining the final answer or not being able to determine the final answer. So, from the last answer of H₁, the error is categorized as EE. To find out more about EE by H₁ in the answer to question number 2e, the following interview was conducted.

R : Question numbers 2a, 2b, 2c, and 2d look like you can answer correctly, why did you make a mistake on problem 2e?

H₁ : I hesitate to use the relationship formula between the solution function $F(x,y)$ and the ODE coefficients $M_1(x,y)$ and $N_1(x,y)$.

R : What made you forget about the formula?

H₁ : Because in the part where I made the mistake, there are two formulas that can be used. Firstly, the relationship formula of $F(x,y)$ with $M_1(x,y)$ and secondly the relationship formula of $F(x,y)$ with $N_1(x,y)$. The formulas are similar so they are often confused when using them.

R : If you were asked to do it again now, would you remember the formula?

H₁ : I remember, sir. (then showed the formula by writing it down)

From the test results and interviews with H₁ presented, it is believed that H₁ did EE. The cause of the error was that H₁ could not use the algorithm correctly and did not master the concept of exact differential equations.

Medium Ability Subjects (M₁)

In question number 1a subject M₁ can solve the problem well. In terms of SOLO taxonomy, it means that M₁ doesn't make mistakes on pre-structural level questions. According to NEA, M₁ didn't make RE. However, M₁ made mistakes on questions number 1b, 1c, 1d, and 1e, namely questions with uni-structural, multi-structural, relational, and extended abstract levels. In this case, the errors made by M₁ are CE, TE, PE, and EE. To confirm the errors and find out the factors that caused the mistakes made by subject M₁, the following answer snippets and interviews are shown.

$$\begin{aligned}
 M(x,y) &= y \sqrt{x^2 + y^2} \\
 M(tx,ty) &= ty \sqrt{(tx)^2 + (ty)^2} \\
 &= ty \sqrt{t^2(x^2 + y^2)} \\
 &= t^2 y \sqrt{x^2 + y^2} \\
 &= t^2 M(x,y) \rightarrow \text{homogeneous function of degree two.} \\
 N(x,y) &= -x(x + \sqrt{x^2 + y^2}) \\
 N(tx,ty) &= -tx(tx + \sqrt{(tx)^2 + (ty)^2}) \\
 &= -tx(tx + t\sqrt{x^2 + y^2}) \\
 &= t(-x(x + \sqrt{x^2 + y^2})) \\
 &= t N(x,y) \rightarrow \text{homogeneous function of degree one.}
 \end{aligned}$$

Since the homogeneous $M(x,y)$ and $N(x,y)$ do not have the same degree it means that the ODE is not homogeneous.

Figure 3. M₁'s answers for questions 1b, 1c, 1d, and 1e

In Figure 3, it can be seen that M_1 cannot show that the ODE is homogeneous. It is shown from M_1 's answer which cannot use a formula to test whether a function is homogeneous or not. The problem experienced by M_1 is in accordance with the indicators of NEA, namely incorrectly determining the formula, determining the steps to solve the problem, and operating calculations in solving the problem. In this case, the errors experienced by M_1 are CE and TE. In addition, errors were also found in the systematisation of problem solving and determining the final answer. These last two errors arise from the occurrence of CE and TE. So, PE and EE also occurred here. To find out more about the mistakes made by M_1 in questions 1b, 1c, 1d, and 1e, the following are excerpts of the researcher's interview (R) with M_1 .

- R : Your answer to question 1 is correct only for number 1a. What were your difficulties in answering question 1b?*
- M_1 : I thought that because the answer to question 1a shows that ODE is not as different as differential equations, there is no need to answer question 1b and I can answer question 1c directly.*
- R : Ok, why is your answer to question 1c also wrong?*
- M_1 : I forgot the formula used to show whether a function is homogeneous or not, so I could not determine whether the given ODE was homogeneous or not.*
- R : Did you know that questions 1a to 1e have to be done in order?*
- M_1 : (Long thought before answering)
I know, sir.*
- R : Because you have made a mistake in the answer to question 1b, the answers to questions 1c to 1e will also be wrong.*

From the test results and interviews with M_1 presented previously, it was found that M_1 did CE, TE, PE, and EE. The causes of these errors were that M_1 didn't write down what was known and what was asked in the problem, forgot the steps to solve the problem, incorrectly determined the integral result, and was wrong in writing the final answer because he was not used to writing the final answer.

Next, we will analyze M_1 's mistakes in problem number 2. In problem numbers 2a, and 2b subject M_1 can solve all the problems well. In terms of SOLO taxonomy, it means that M_1 didn't make mistakes on pre-structural, and uni-structural level questions. According to NEA, M_1 didn't make RE. However, M_1 made errors in questions number 2c, 2d, and 2e with multi-structural, relational, and extended abstract levels, namely TE, PE, and EE. To confirm the errors and find out the factors that caused the errors made by subject M_1 in the question, the following are shown snippets of answers and interviews.

$$M(x) [(3x^2 + y^2) dx - 2xy dy] = 0 \quad dy$$

$$\frac{1}{x^2} [(3x^2 + y^2) dx - 2xy dy] = 0$$

$$\left(3 + \frac{y^2}{x^2}\right) dx - \frac{2y}{x} dy = 0$$

Thus $M_1(x,y) = 3 + \frac{y^2}{x^2} \rightarrow \frac{\partial M(x,y)}{\partial y} = \frac{2y}{x^2}$

$N_1(x,y) = -\frac{2y}{x} \rightarrow \frac{\partial M(x,y)}{\partial x} = \frac{2y}{x^2}$

The equation is exact.

$$\frac{\partial F(x,y)}{\partial x} = N_1(x,y) \Rightarrow F(x,y) = \int \left(-\frac{2y}{x}\right) dx$$

Figure 4. M₁'s answers for questions 2c, 2d, and 2e

Figure 4 showed that M₁ had problems in determining the steps of solving the problem and using the formula. M₁ was also incorrect in determining the integral result that appeared in the answer. The error occurred in using the relationship between the solution function F(x,y) and the ODE coefficients M₁(x,y) and N₁(x,y), so that the integral calculation performed didn't produce a general solution of the ODE. The problem experienced by M₁ is in accordance with the indicators of NEA, namely forgetting the steps of solving the problem, incorrectly determining the integral result, and incorrectly determining the final answer. From the final answer of M₁, the errors made were categorized into TE, PE, and EE.

R : Question numbers 2a and 2b look like you can answer correctly, why did you make mistakes on questions 2c, 2d, and 2e?

M₁ : I hesitated to use the integral methods.

R : What makes you doubt the integral calculation?

M₁ : Because the integrating technique has a lot of formulas to remember.

R : If you were asked to do it again now, would you remember the formula you used?

M₁ : By looking back at my notebook, I think I can do it, sir.

(tried to work on it, although the results were not complete)

From the test results and interviews with M₁ presented above, it is believed that M₁ did TE, PE, and EE. The cause of the error is that M₁ cannot use the algorithm correctly and master the concept of exact differential equations.

Low Ability Subjects (L₁)

Subject L₁ made mistakes in all problems from 1a to 1e. No problem was done correctly. In terms of SOLO taxonomy, it means that L₁ made mistakes at all levels of the problem starting from the pre-structural level to the extended abstract level. According to NEA, L₁ did RE to EE. To confirm the errors and find out the factors that caused the mistakes made by subject L₁, the following answer snippets are shown followed by an interview.

$$\begin{aligned}
 &M(x,y)dx + N(x,y)dy = 0 \\
 &y\sqrt{x^2+y^2}dx - x(x+\sqrt{x^2+y^2})dy = 0 \\
 &(a) \quad M(x,y) = y\sqrt{x^2+y^2} \\
 &\quad \quad \quad = y(x^2+y^2)^{1/2} \\
 &\quad \quad \quad N(x,y) = -x(x+\sqrt{x^2+y^2}) \\
 &\quad \quad \quad = -x^2 - x\sqrt{x^2+y^2} \\
 &\quad \quad \quad = -x^2 - x(x^2+y^2)^{1/2} \\
 &\therefore \text{It is obtained that } M(x,y) \text{ and } N(x,y) \\
 &\text{as homogeneous functions the same} \\
 &\text{degree, namely degree } 1/2.
 \end{aligned}$$

Figure 5. L₁'s answers for questions 1a to 1e

Figure 5 showed that L₁ had problems in reading the problem and didn't understand the meaning of the problem and could not take the keywords from the problem command. As a result, the answers to each level of the problem didn't appear. It is because from the initial stage there were errors, so it didn't support writing the next answers.

R : In your answer to question 1 it appears that all the questions cannot be answered correctly, why is this?

L₁ : I don't understand this ODE material, sir.

R : What efforts did you make to understand this ODE material?

L₁ : I admit that the effort to understand this is lacking, sir.

R : Don't you want to be able to understand this ODE material like most of your other friends?

L₁ : Frankly, I really want to sir. But, I don't know where to start. Moreover, I am not very familiar with integral material.

R : Ok, keep up the good work. Try to find a friend to study with.

From the test results and interviews with L₁, it can be concluded that L₁ really didn't understand the questions tested, so the answers she wrote didn't lead to the answers to the questions. The reason is that L₁ is weak in integral material. Even though this integral is a prerequisite for this ODE course. As a result, L₁ was confused about where to start to understand the ODE material.

Furthermore, in analyzing L₁'s mistakes in problem number 2, the mistakes she made were the same as those in problem number 1, where questions 2a to 2e were done incorrectly. If continued with the interview, it is certain that the answer will be the same as the interview results for his answer to question number 1 before. So, the answer to question number 2 is not continued with an interview. In conclusion, the types of errors and their causes are the same as what happened when L₁ worked on problem number 1.

3.2. Discussion

The study found all NEA components in students' answers. It doesn't rule out the possibility for students to make mistakes at each level of questions adjusted to SOLO taxonomy. Pre-structural or uni-structural levels in SOLO taxonomy are more prone to RE and CE in NEA. At the multi-structural level, TE and PE are more common as students begin to apply the solution method but have not fully mastered it. The relational level tends to have fewer errors, but can still experience EE when writing the final solution. Meanwhile, at the extended abstract level, students are less likely to make mistakes because they already understand the concept deeply and flexibly. Through the results of this analysis, the causes of the errors made by these students can be known. The study of Kusmaryono et al. (2018) revealed the process of changing concrete reasoning to abstract. The quantitative stage (uni-structural and multi-structural) occurs first, such as the number of student responses increases, then undergoes qualitative changes (relational and extended abstract) because it has been integrated into a structural pattern.

RE arises when students cannot read keywords or symbols in the problem (Mubarokah & Amir, 2024; Yarman et al., 2024). RE can be identified through the interview process. In this study, there were four students who made REs. This is in line with previous findings that RE is caused because students are wrong in illustrating the problem into the expected mathematical sentence. Such as Mubarokah and Amir (2024) which revealed 68.57% of students could not analyse the sentence 'can cut and use some tiles' in the problem so that they were wrong in drawing a sketch. In addition, 8.57% of students could determine formulas and calculation operations but made mistakes in interpreting symbols. On the other hand, Yarman et al. (2024) revealed that RE is a rare case. In his study, only 1.49% of students made RE in solving the double integral. Wardhani and Argaswari (2022) research also revealed that no RE was found in the answers of high school students in solving trigonometry story problems.

CE is the next type of error after students can read well but do not know what problems to solve (Wardhani & Argaswari, 2022). Students do not write down what is known and asked in the problem because they are not used to writing it down and are incomplete in writing it due to students' inaccuracy when reading the problem. Relevant to Wardhani and Argaswari (2022) findings where 17.77% of students made CE because they did not write and understand the elevation angle which is an important information component in the problem. Sometimes students intend to shorten the sentence, but the writing is not in accordance with the meaning of the question. This happened because they were in a hurry to do the problem. About 1.49% of students made CE in determining the point of intersection of lines and integral boundaries (Yarman et al., 2024). CE was also found by Mubarokah and Amir (2024) which 28.57% of students understood the term "many tiles that can cover the entire surface of the bedroom floor". They directly multiplied the bedroom area by the tile area in determining the number of tiles. Furthermore, 91.43% of students were wrong in converting metres to centimetres.

The third error is TE which occurs when students cannot determine mathematical operations or procedures appropriately. This can be seen from the answers of students who are not able to use arithmetic operations correctly or use formulas that are in accordance with

the demands of the problem. The cause is that students do not make mathematical models because they do not feel the need to write them down. As Mubarokah and Amir (2024) found, 11.43% of students were wrong in drawing a bedroom sketch because they did not pay attention to the question instructions “the tiles arranged should not have gaps and should not overlap”. About 82.86% of students used the formula first instead of sketching the bedroom. The error in determining the calculation operation is very influential on PE and EE. Although students can do the calculation correctly, if the operation used is wrong then the result is still wrong. Yarman et al. (2024) also found a similar thing where 46.27% of students didn’t draw the R area first so that it affected the integral boundary and the result. Not only that, low ability students are often wrong in determining the strategy used in solving problems (Wardhani & Argaswari, 2022).

Furthermore, PE occurs because students fail to use the algorithm sequentially and correctly. Students' inaccuracy when working on problems causes errors in the calculation process (Mubarokah & Amir, 2024). Wardhani and Argaswari (2022) found 29.8% of students were wrong in determining the value of $\cos 30^\circ$ so that the final answer was wrong. PE became the dominant error in performing algebraic calculation operations to solve the twofold integral (Yarman et al., 2024). Finally, students didn’t make a conclusion or final result which is categorised as EE. Almost most students made this mistake, which is in line with Wardhani and Argaswari (2022) findings. It is because students have been wrong in the previous calculation process and are not used to writing conclusions (Mubarokah & Amir, 2024). However, it is different from the findings of Yarman et al. (2024) which no EE was found in his students' answers.

Not only fixated on NEA, Ekamornaroon et al. (2024) identified common errors of junior high school students in solving polynomial problems. Most students made computational errors in determining similar terms, addition and subtraction of polynomials at 17.86%, 10.88%, and 12.04% respectively. However, careless error was the highest error in polynomial multiplication. He emphasized that the understanding and visualization of whole number operations is the main foundation for students in solving polynomials. As Mahadewsing et al. (2024) argues, students who don’t have sufficient prior knowledge tend to make mistakes in solving mathematics problems. Routine errors will cause cognitive conflict in students (Pratiwi et al., 2022).

4. CONCLUSION

High ability students tend to make PE and EE. One of them made TE and the other one made CE. Unlike the medium ability students, they made CE, TE, PE, and EE. One of them made RE because he could not take keywords and important information from the problem. Meanwhile, low ability students tend to do all NEA components. However, there was one low ability student who did not do RE. Some of the causes of these errors include students’ low ability to read mathematical symbols, students’ inaccuracy, unable to use the algorithm correctly, not mastering the concepts of algebra, differential, and integral, as well as not understanding in determining the systematic solution of the problem and not accustomed to writing the final answer. By understanding the relationship between these two

approaches, educators can develop more effective learning strategies in teaching first-order ODE. Problem-based learning that emphasizes conceptual understanding and systematic practice can help students reach higher levels of understanding in the SOLO taxonomy and reduce errors in the solution process.

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